Subspace Fluid Re-Simulation

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200 x 266 x 200 simulation, 7 hours 2 minutes


Original MacCormack Simulation
Vorticity Confinement
set to zero

Outline

- Previous Work
- Subspace Basics
- The Cubature Approach
- Other Features
- Results
- Discussion and Conclusions

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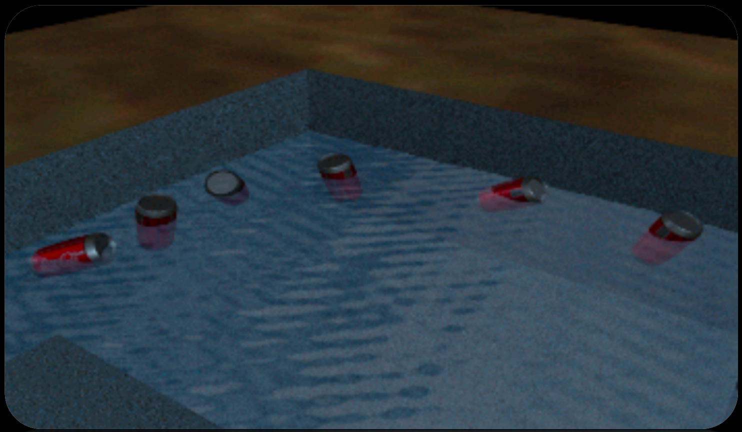
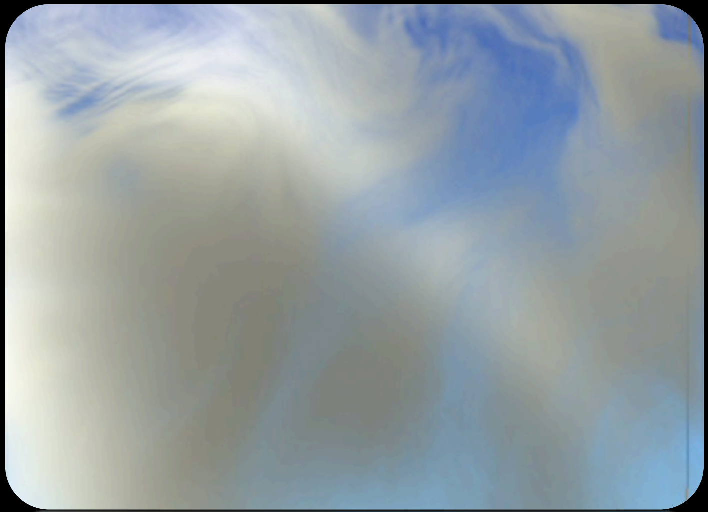
Fluid Simulation



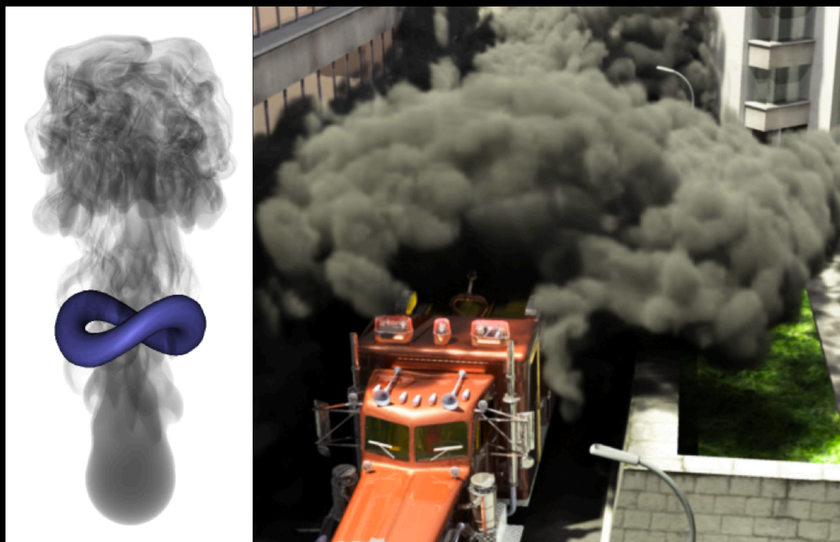
[Foster and Metaxas 1997]



[Stam 1999]

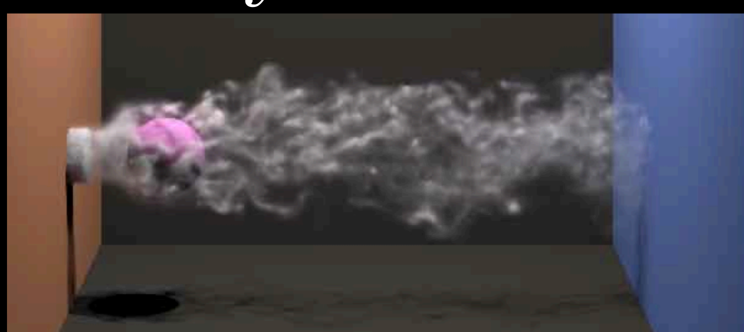
Semi-Lagrangian Advection
Implicit Integration

Vortex Methods



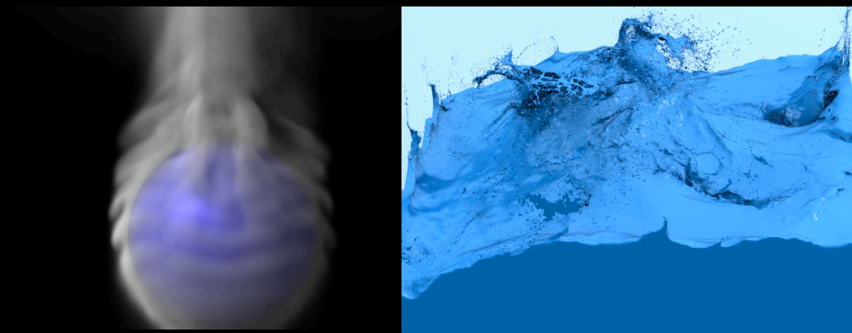
[Brochu et al. 2012]
[Pfaff et al. 2012]

Viscosity Methods



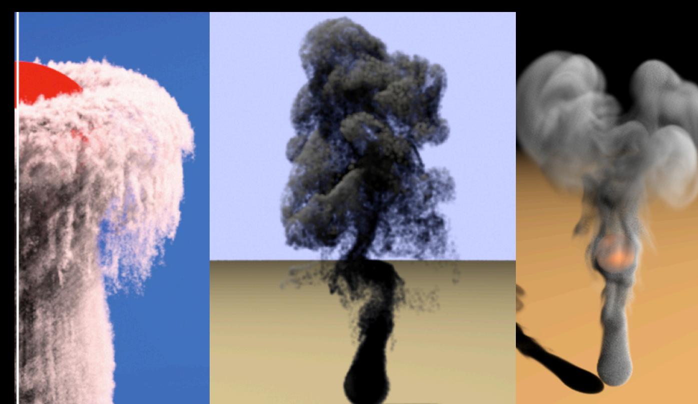
[Mullen et al. 2009]

Adaptive Methods



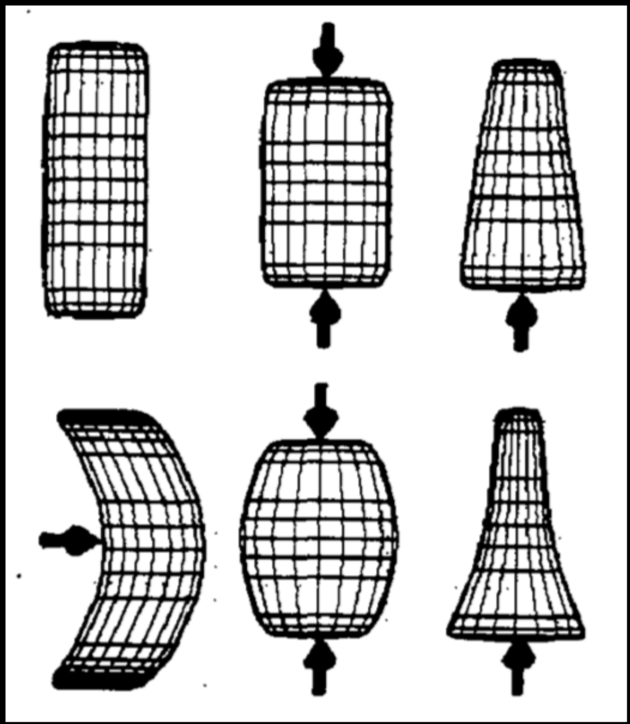
[Losasso et al. 2004]
[Ando et al. 2013]

Turbulence Methods

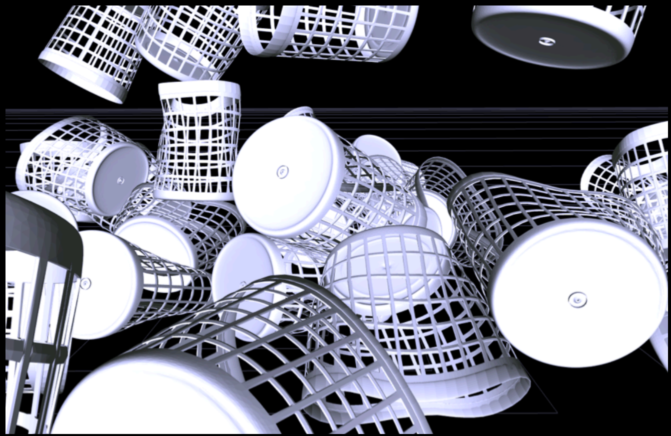


[Kim et al. 2008]
[Schechter and Bridson 2008]
[Narain et al. 2008]

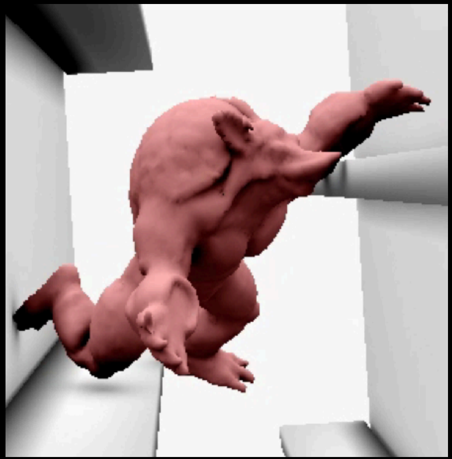
Subspace Methods



[Pentland and Williams 1989]



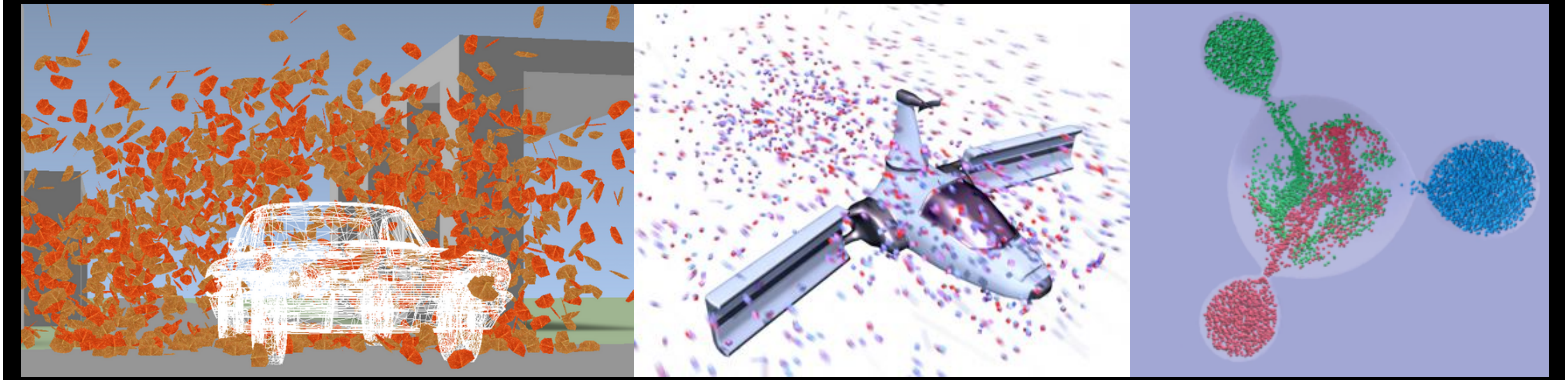
[Barbic and James 2005]



[Kim and James 2011]

[Harmon and Zorin 2013]

Subspace Methods



[Treuille et al. 2006]
[Wicke et al. 2009]
[Stanton et al. 2013]

[Treuille et al. 2006]

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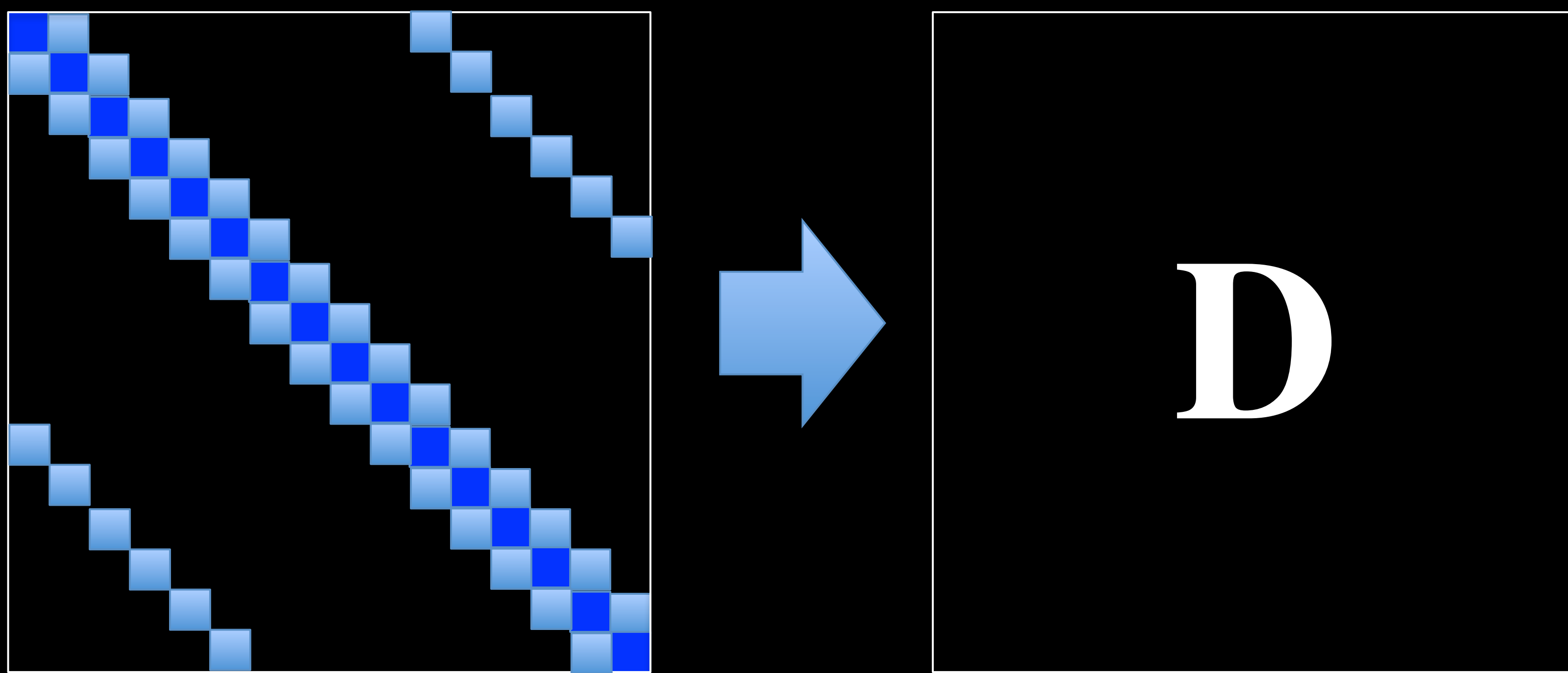
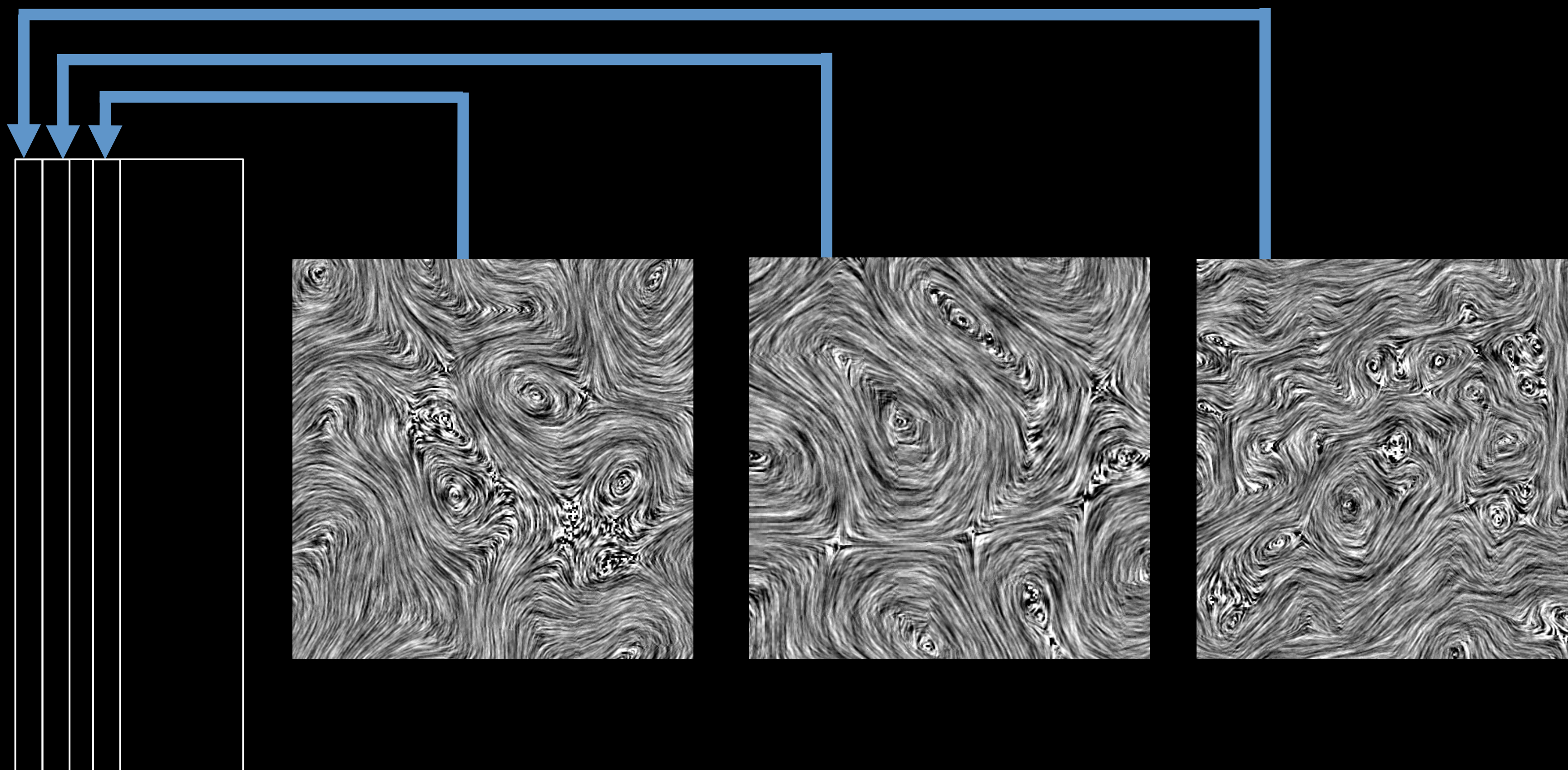
Diffusion



Diffusion



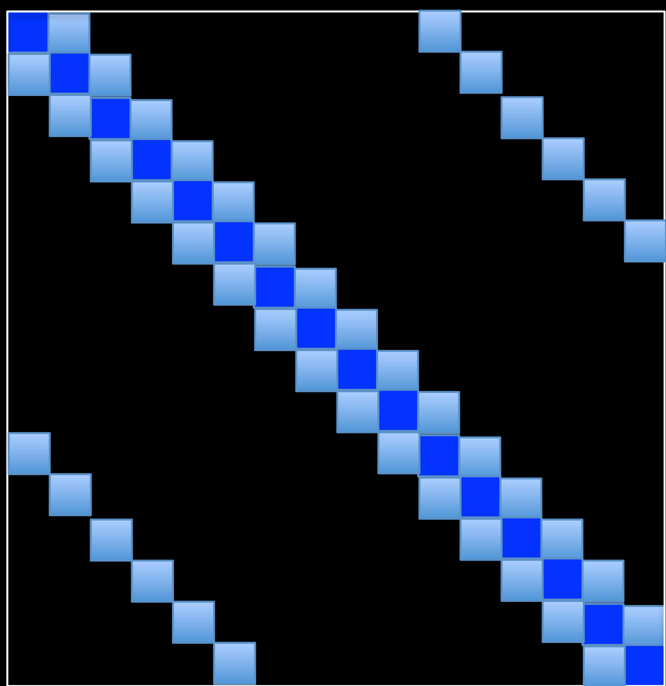
$$\mathbf{U} \in \mathbb{R}^{r \times N} \qquad r \ll N$$

$$\mathbf{U}^T \mathbf{D} \mathbf{U} = \tilde{\mathbf{D}}$$

Stable Fluids

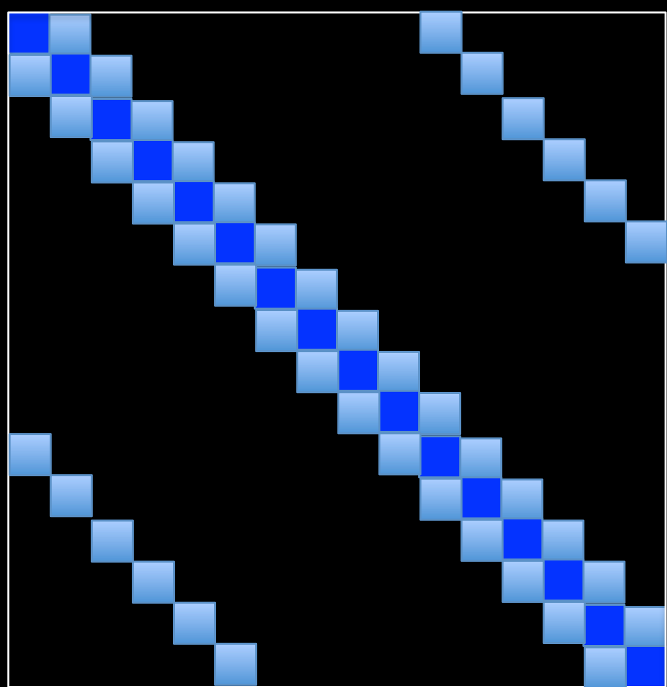
- Diffusion 
- Projection 
- Advection ???

Diffusion



Semi-Lagrangian
Advection

Diffusion

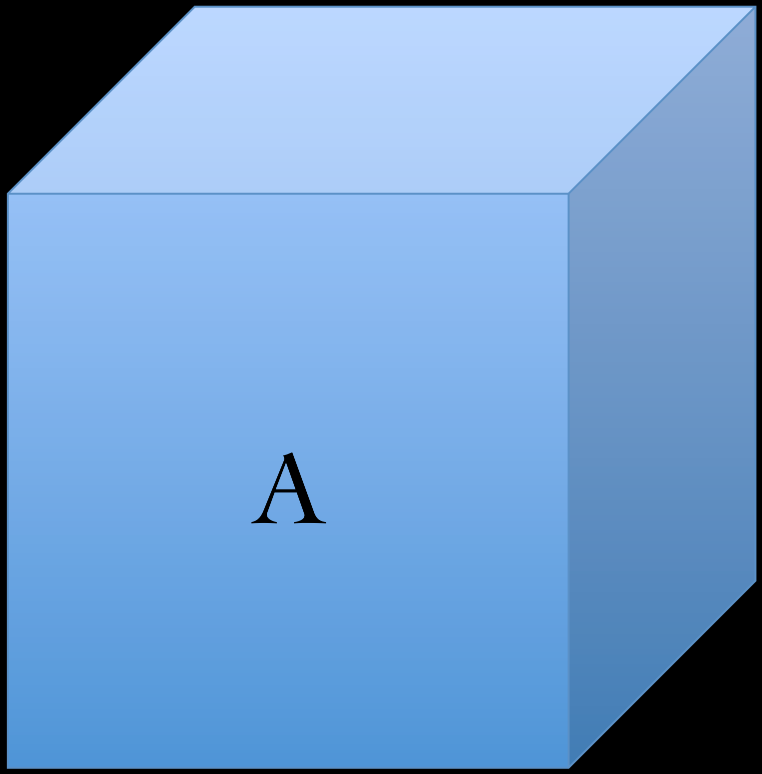


Semi-Lagrangian Advection

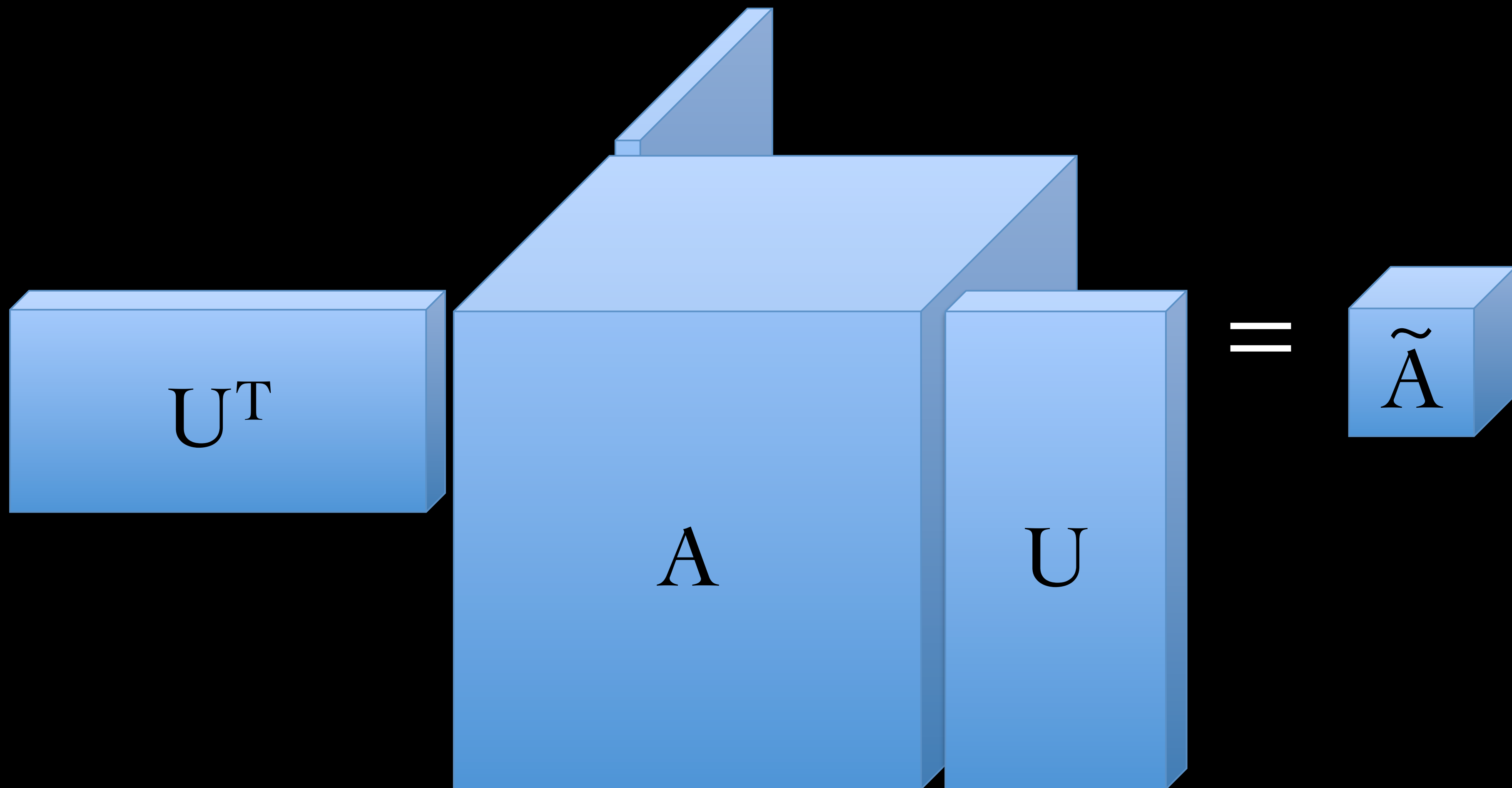
???

$$(\mathbf{u} \cdot \nabla)\mathbf{u}$$

Finite Difference Advection
[Treuille et al. 2006]

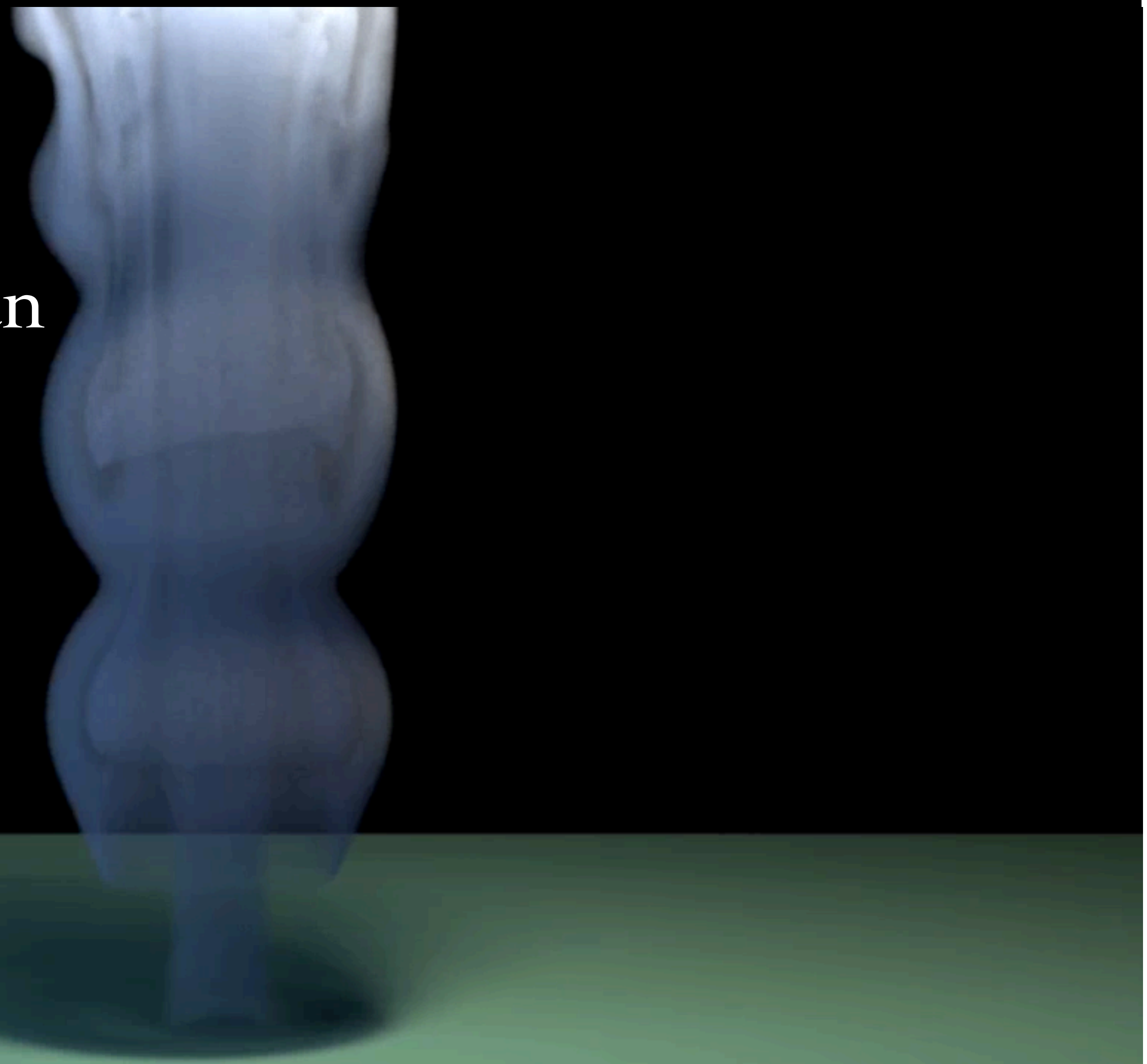


Finite Difference Advection
[Treuille et al. 2006]



Finite differences,
not Semi-Lagrangian

Exponential,
not implicit

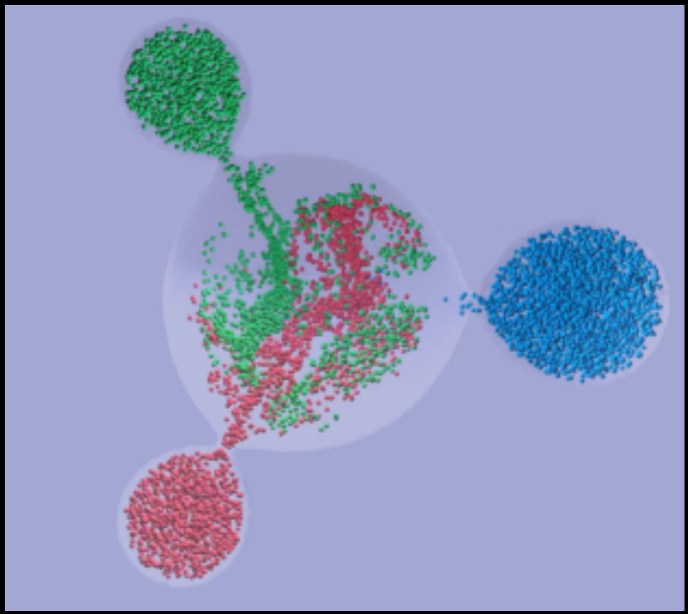


Semi-Lagrangian Advection

???

Semi-Lagrangian Advection

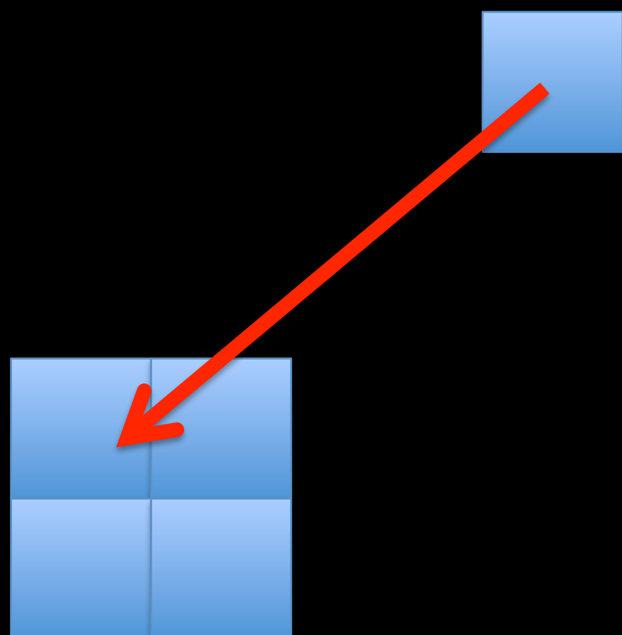
???



[Stanton et al. 2013]

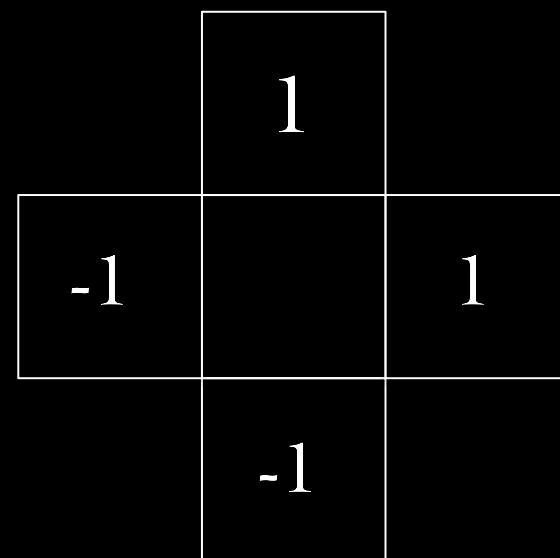
Enables $\div$ and $\sqrt{\ }$

Semi-Lagrangian
Advection



$\neq$

Finite Difference
Advection



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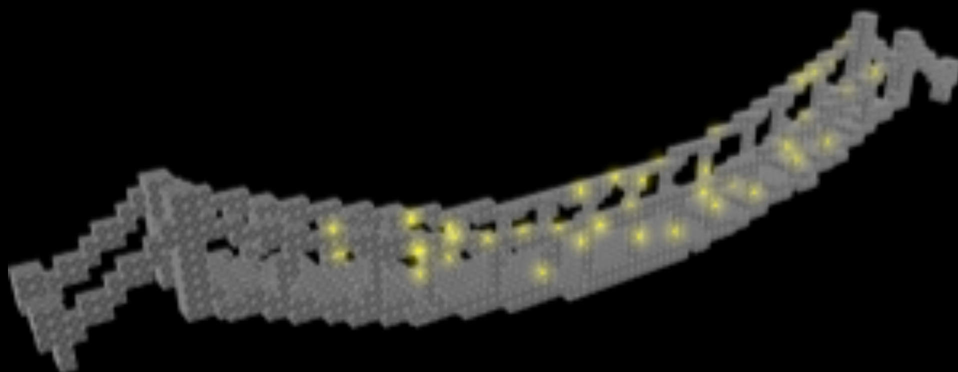
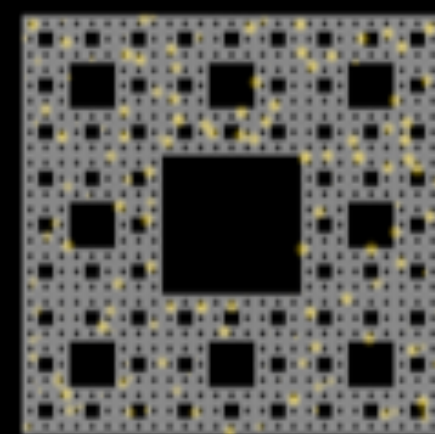
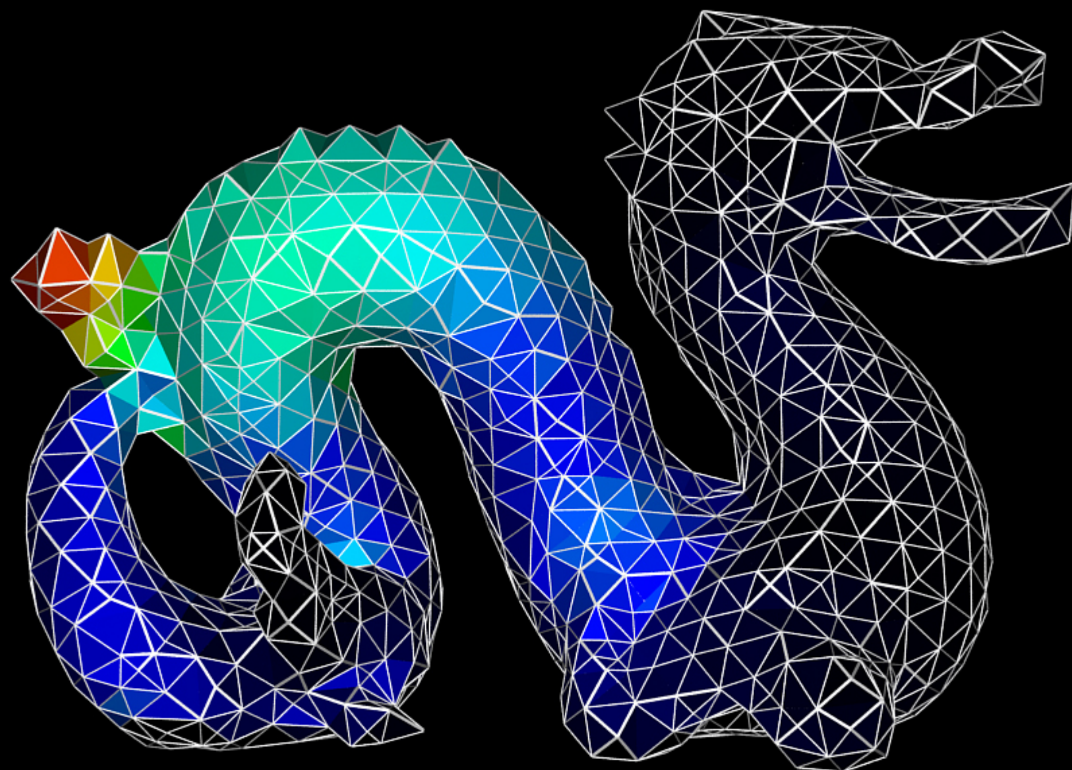
Optimizing Cubature for Efficient Integration of Subspace Deformations [An et al.] 2008



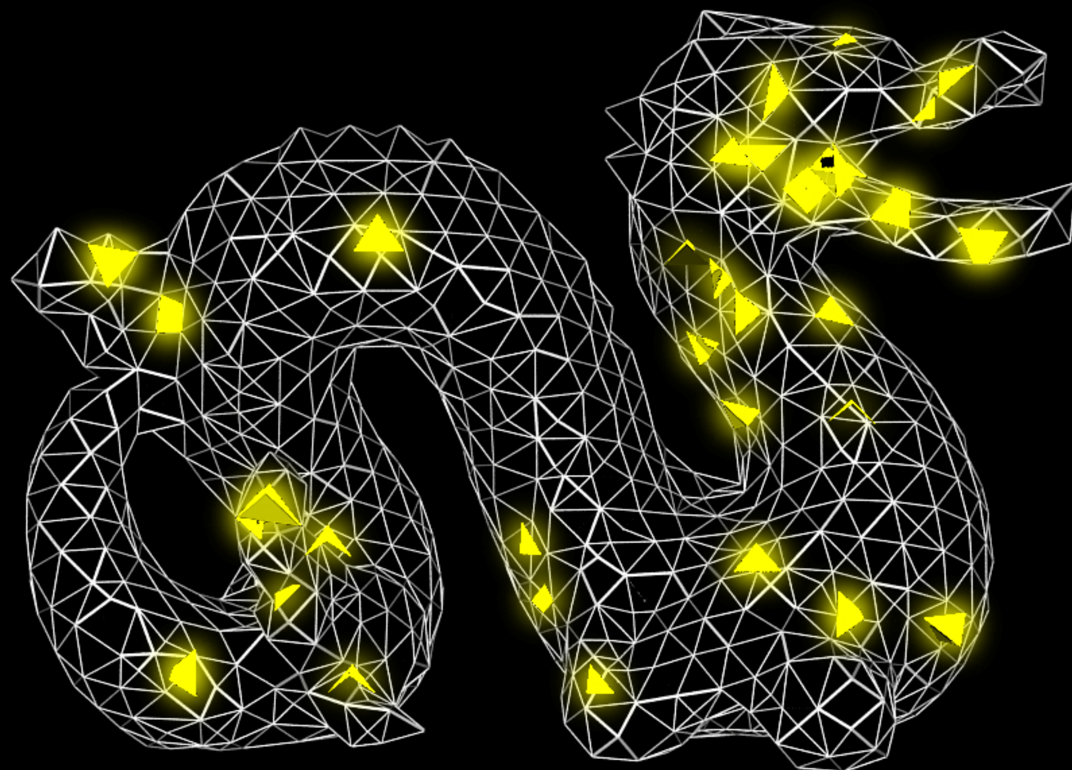


The Cubature Approach

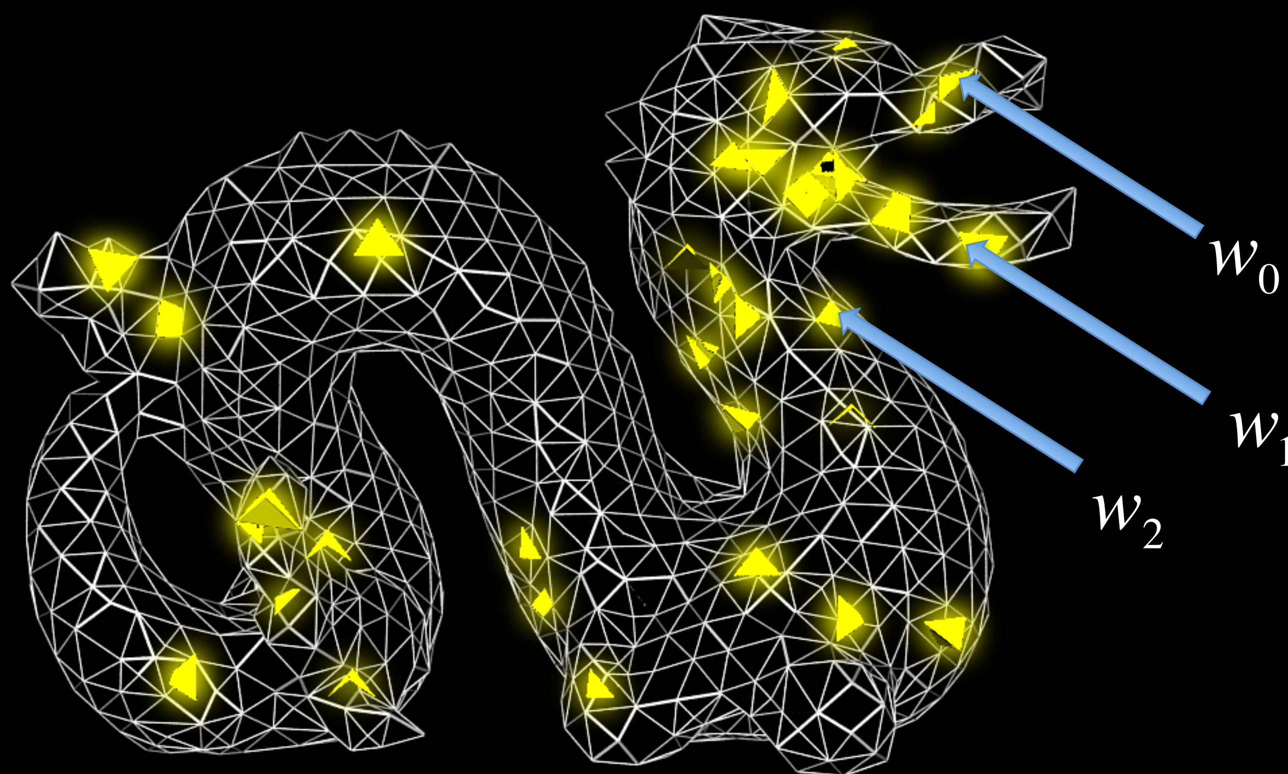


f(x)

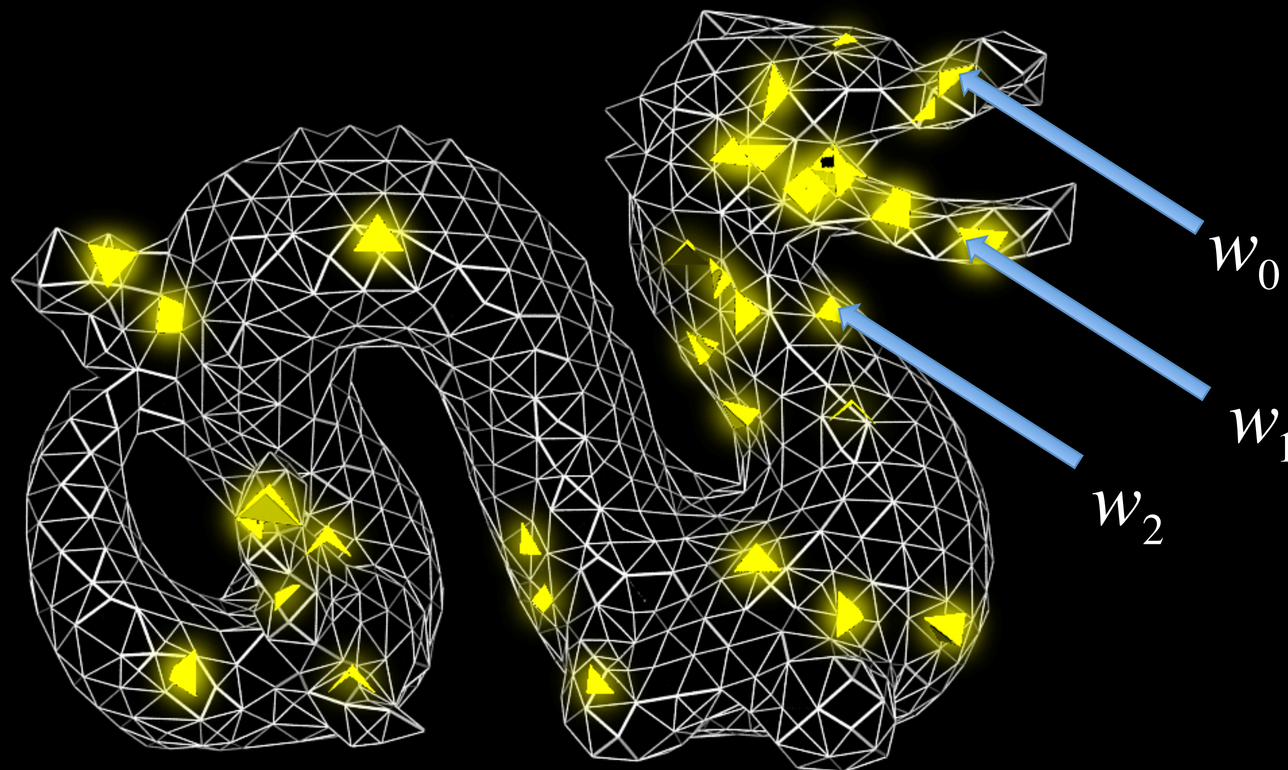
The Cubature Approach



The Cubature Approach



The Cubature Approach



$$\tilde{\mathbf{f}}(\mathrm{x}) \approx \sum_{i=1}^{n} w_i \, \tilde{\mathbf{f}}(\mathrm{x_i})$$

The Cubature Approach

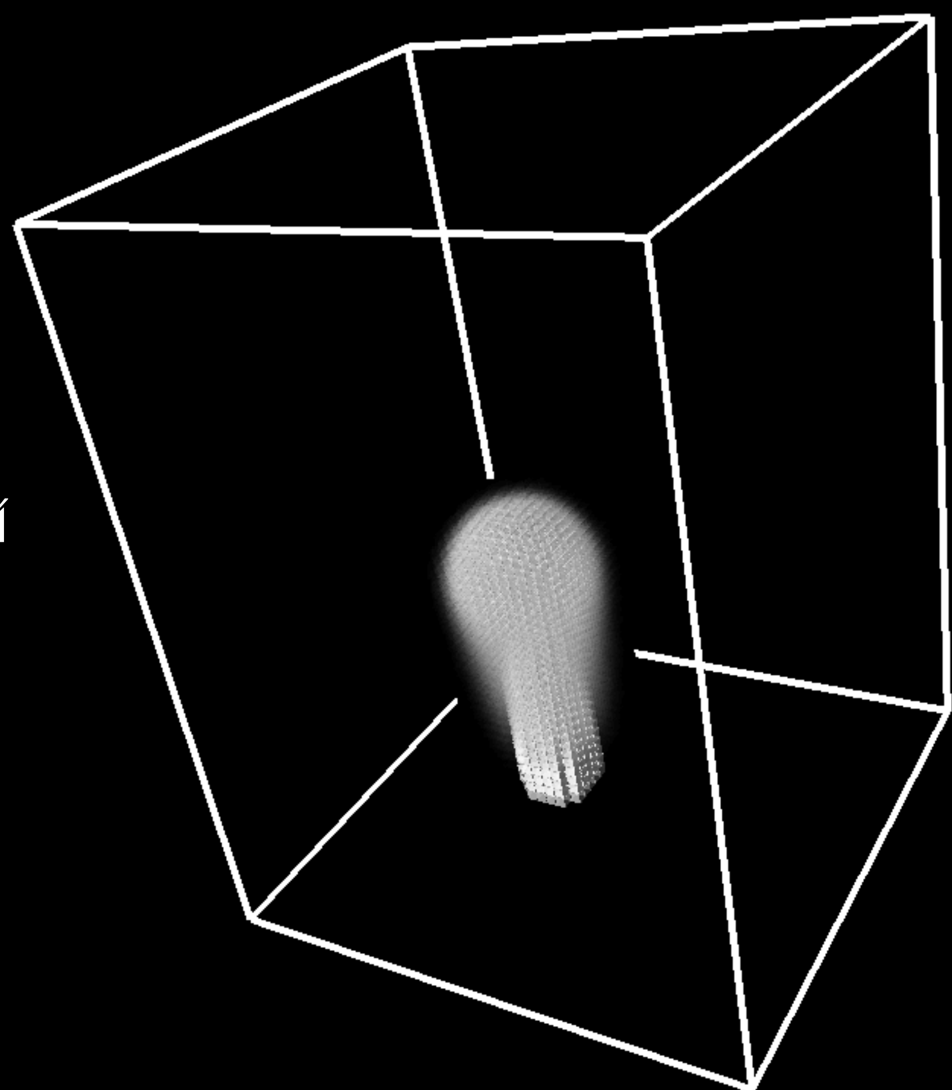
$$\mathbf{f}(x)$$

The Cubature Approach

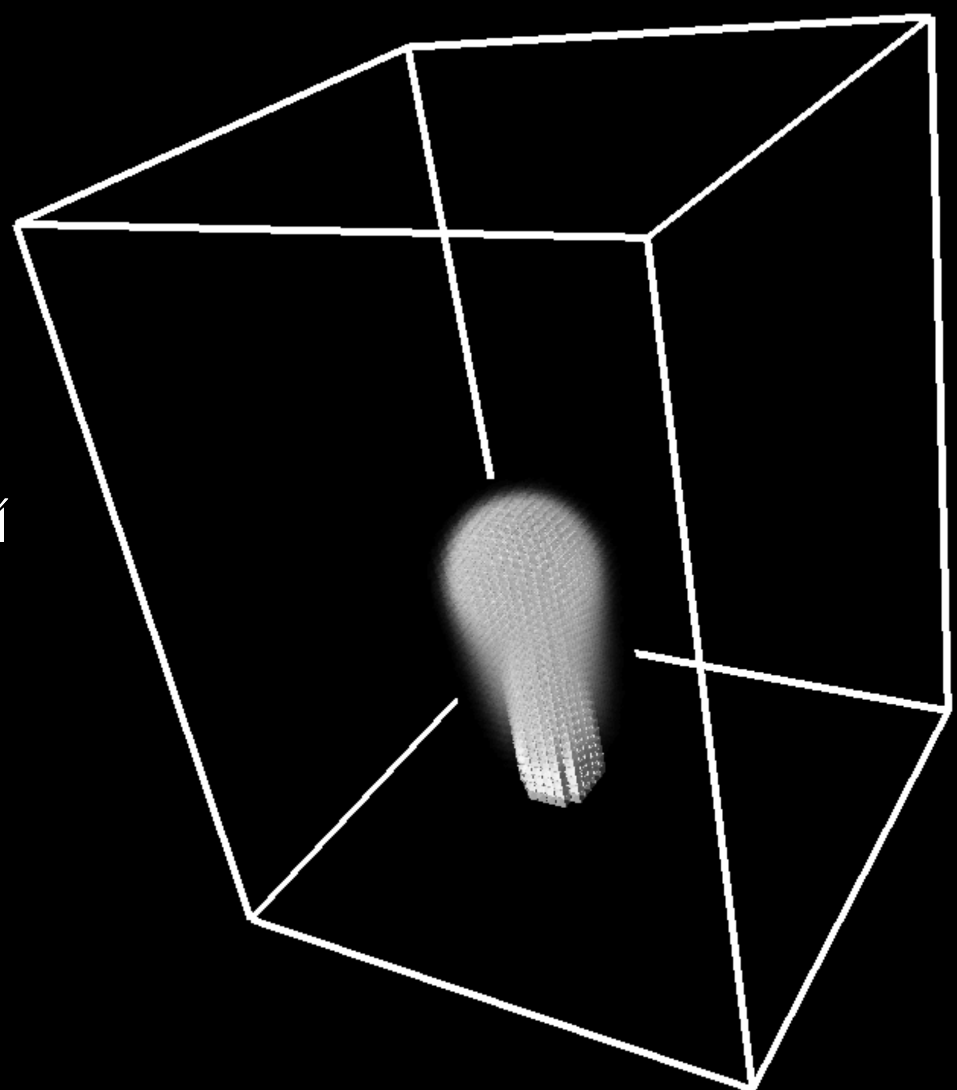
$$\mathbf{f}(x) = \mathbf{A}(\mathbf{u})$$

The Cubature Approach

64 x 48 x 64



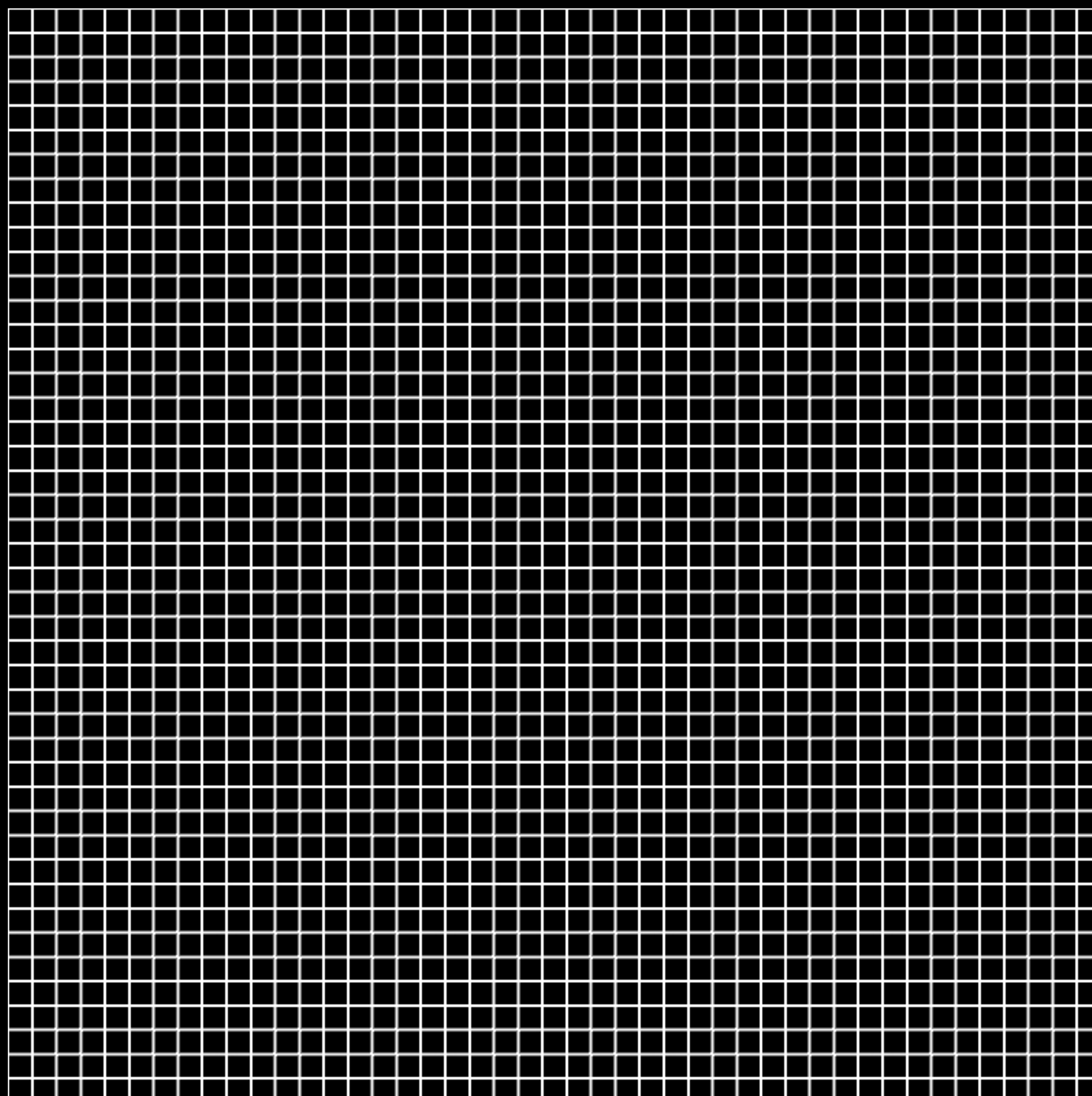
The Cubature Approach



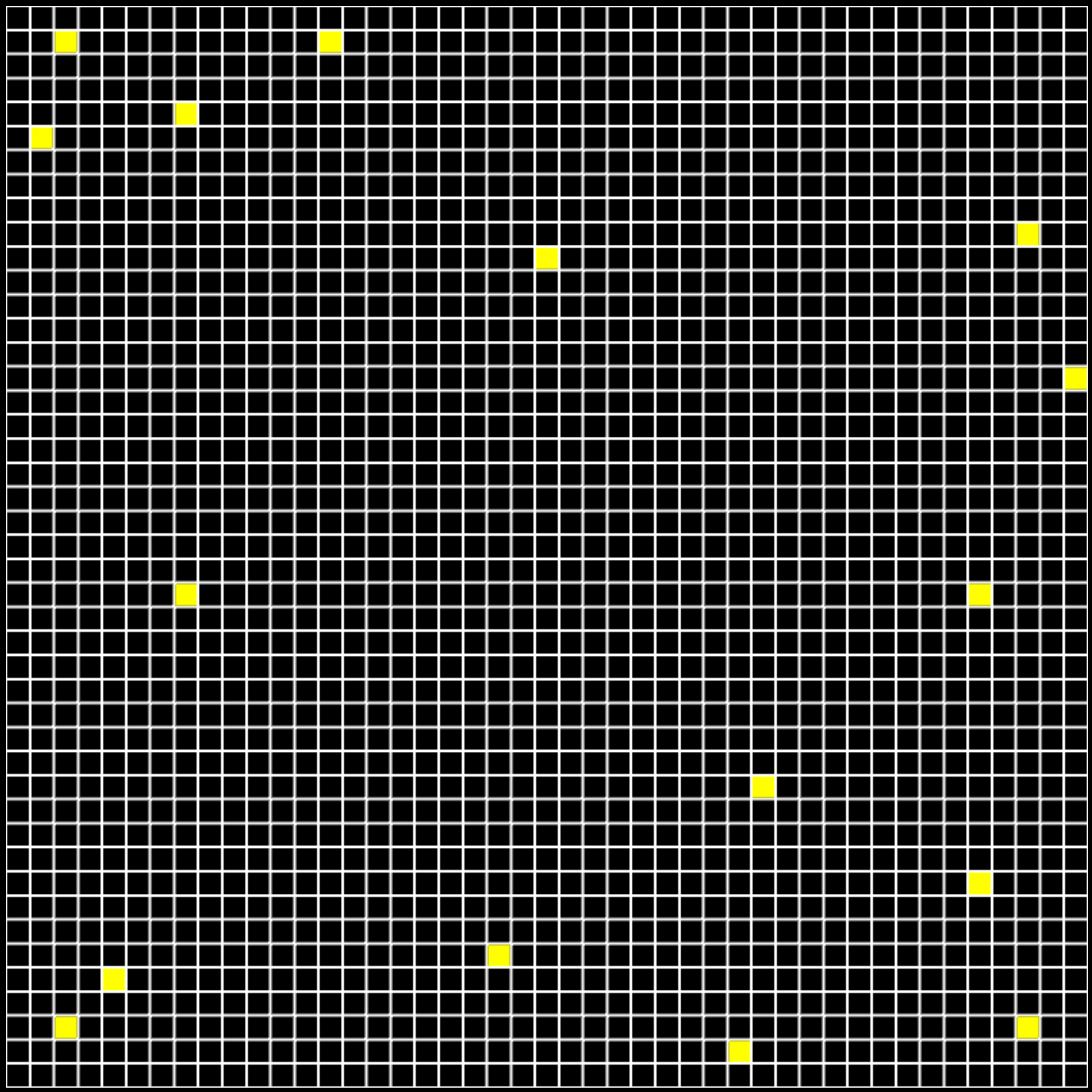
64 x 48 x 64

Six days!

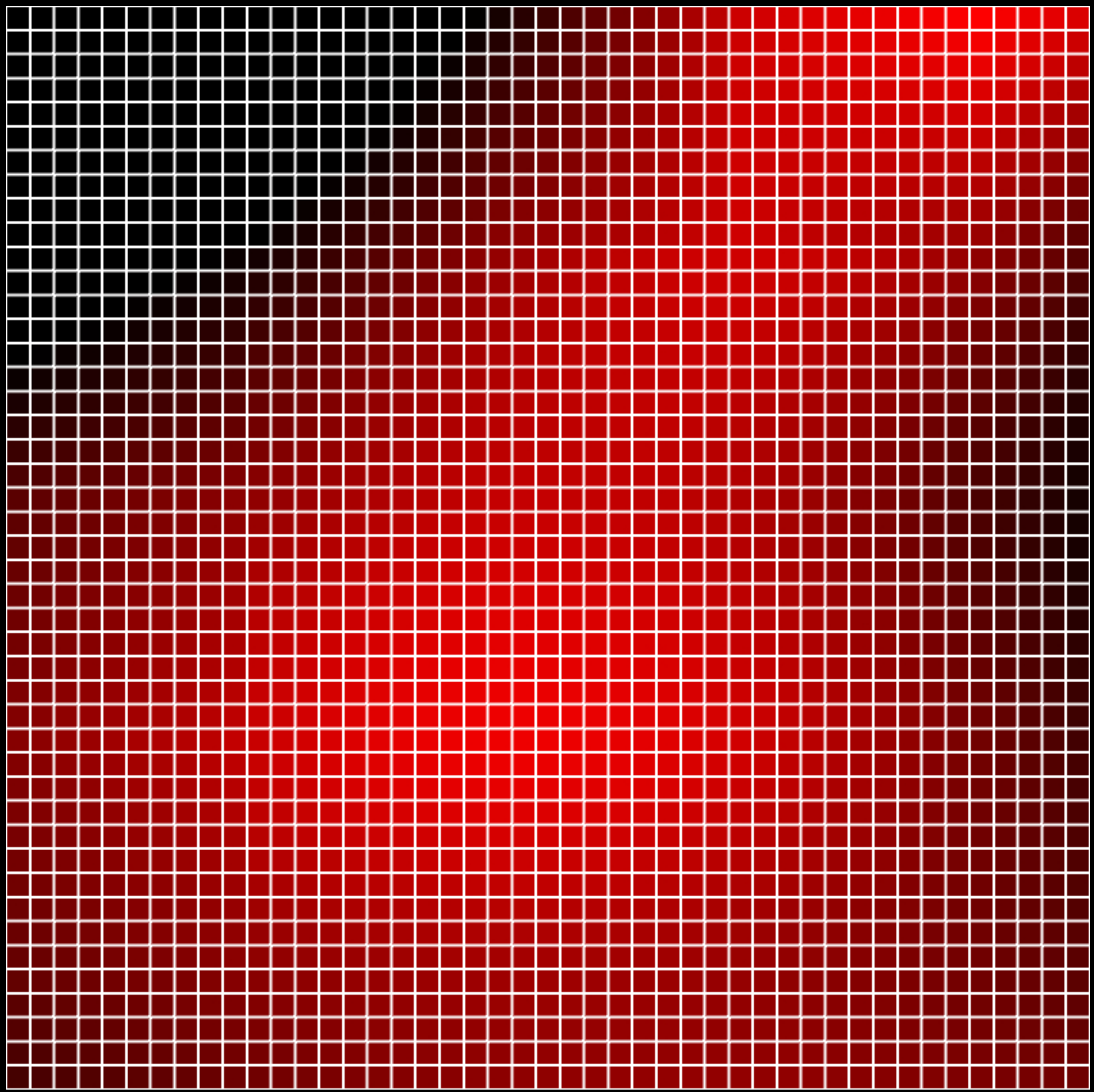
The Greedy Algorithm



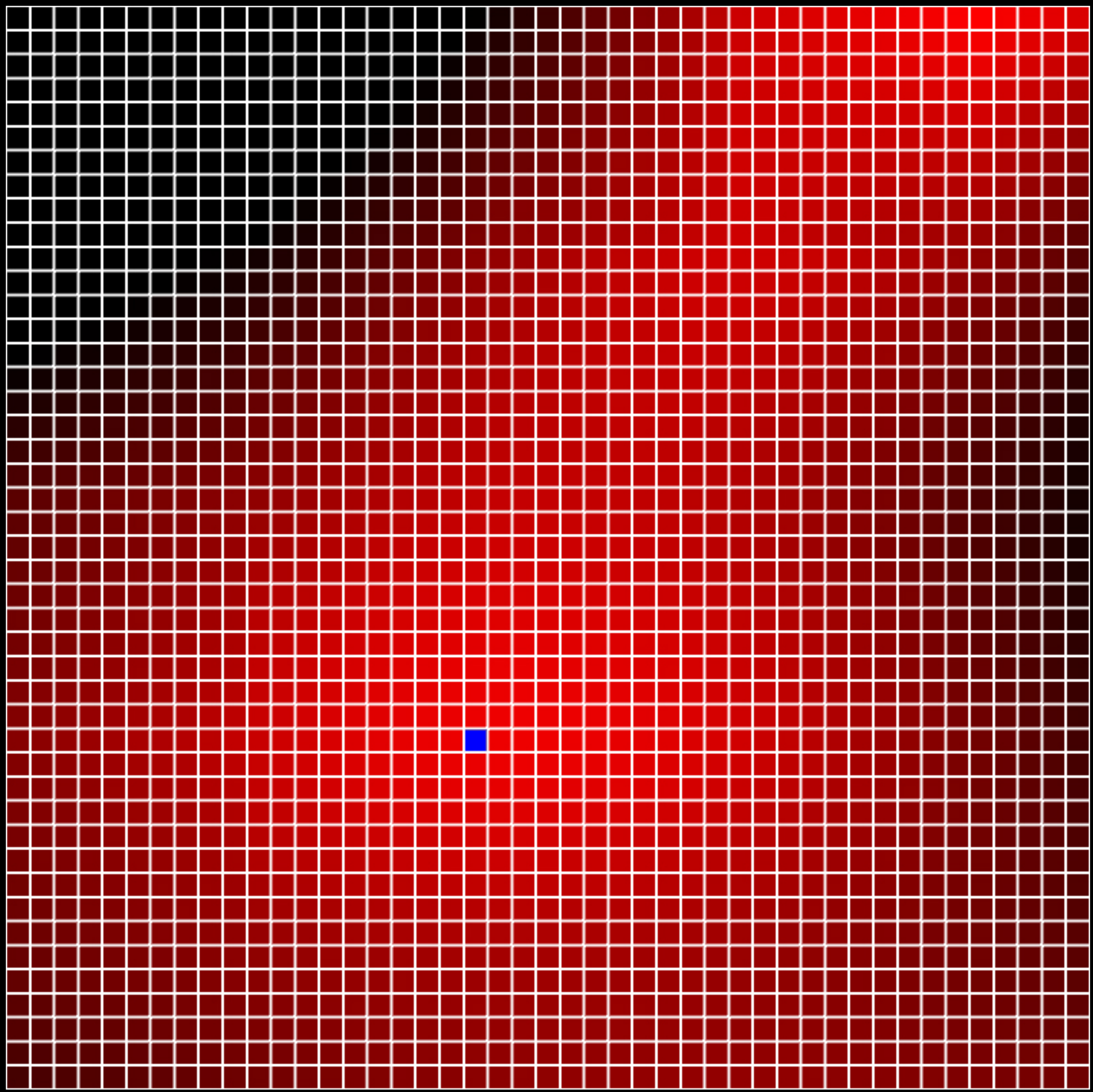
The Greedy Algorithm



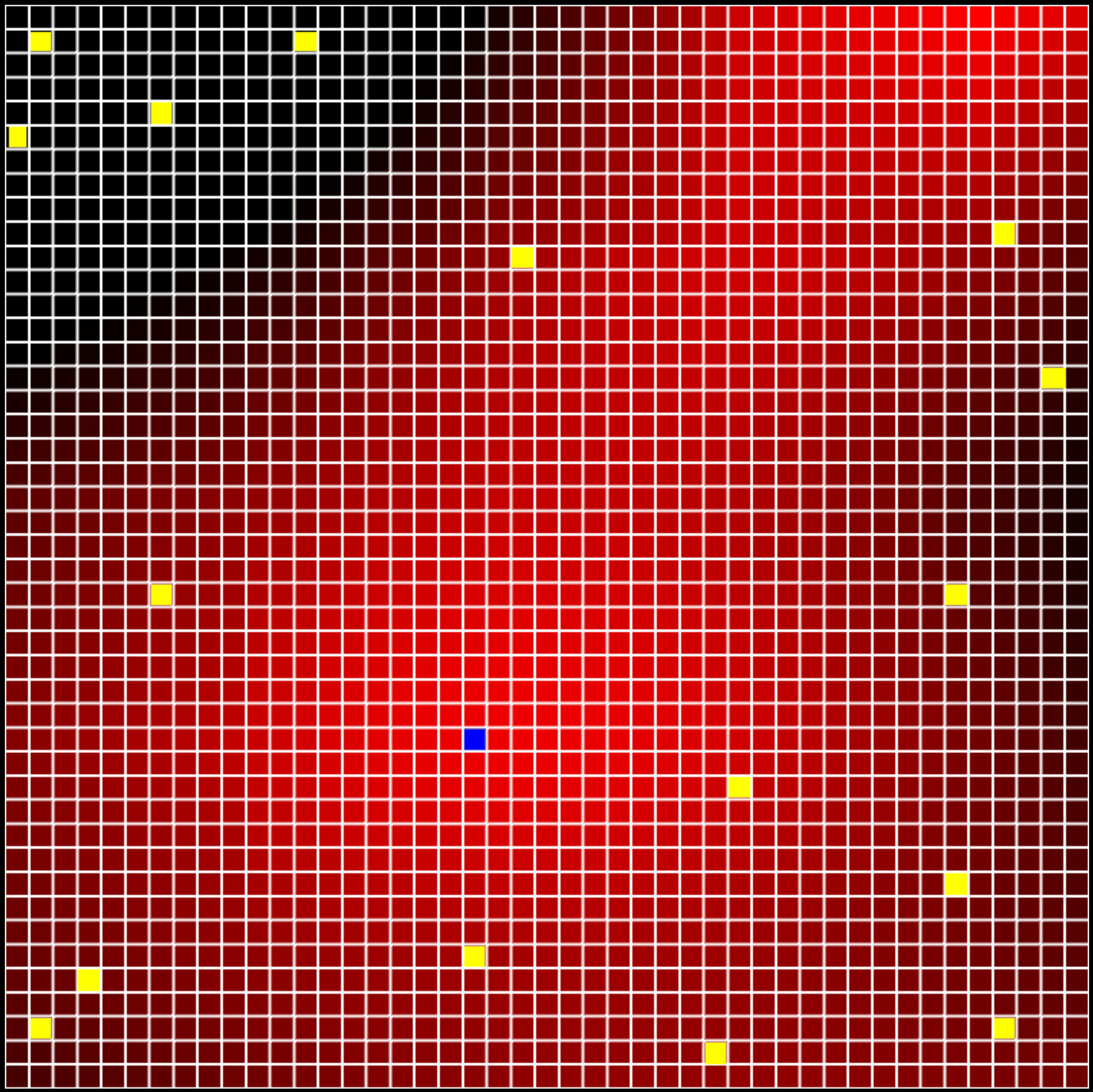
The Greedy Algorithm



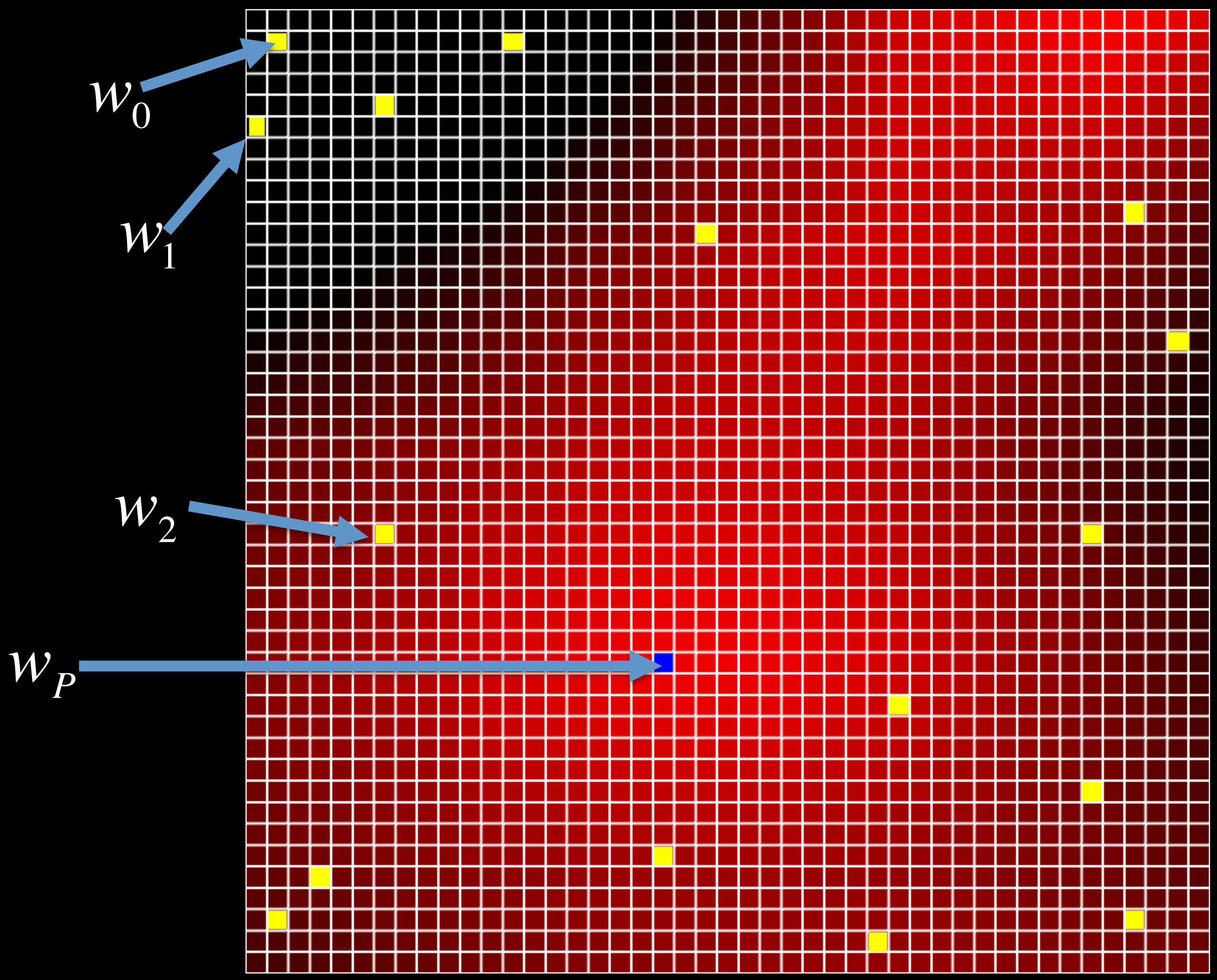
The Greedy Algorithm



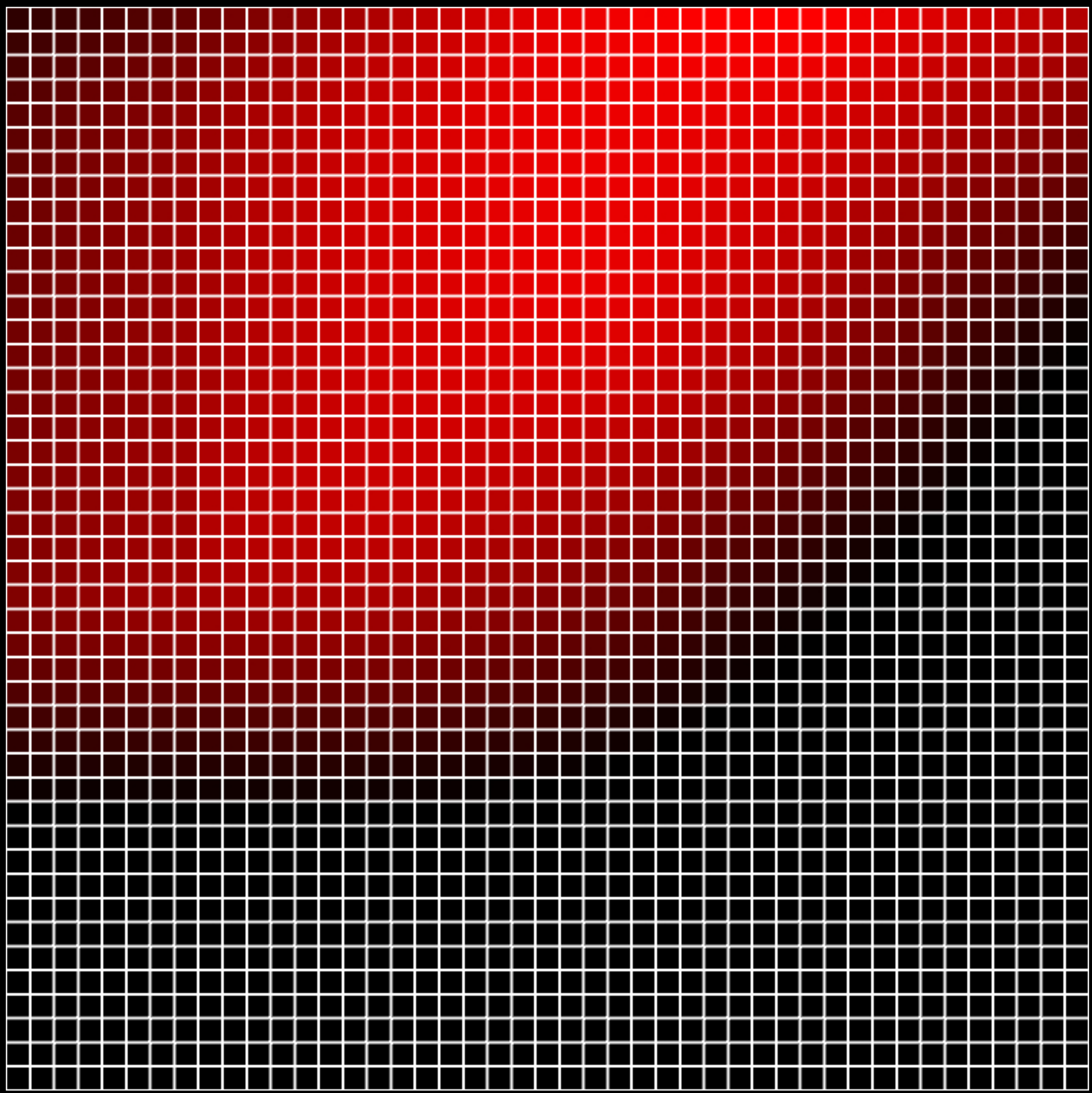
The Greedy Algorithm



Non-Negative Least Squares

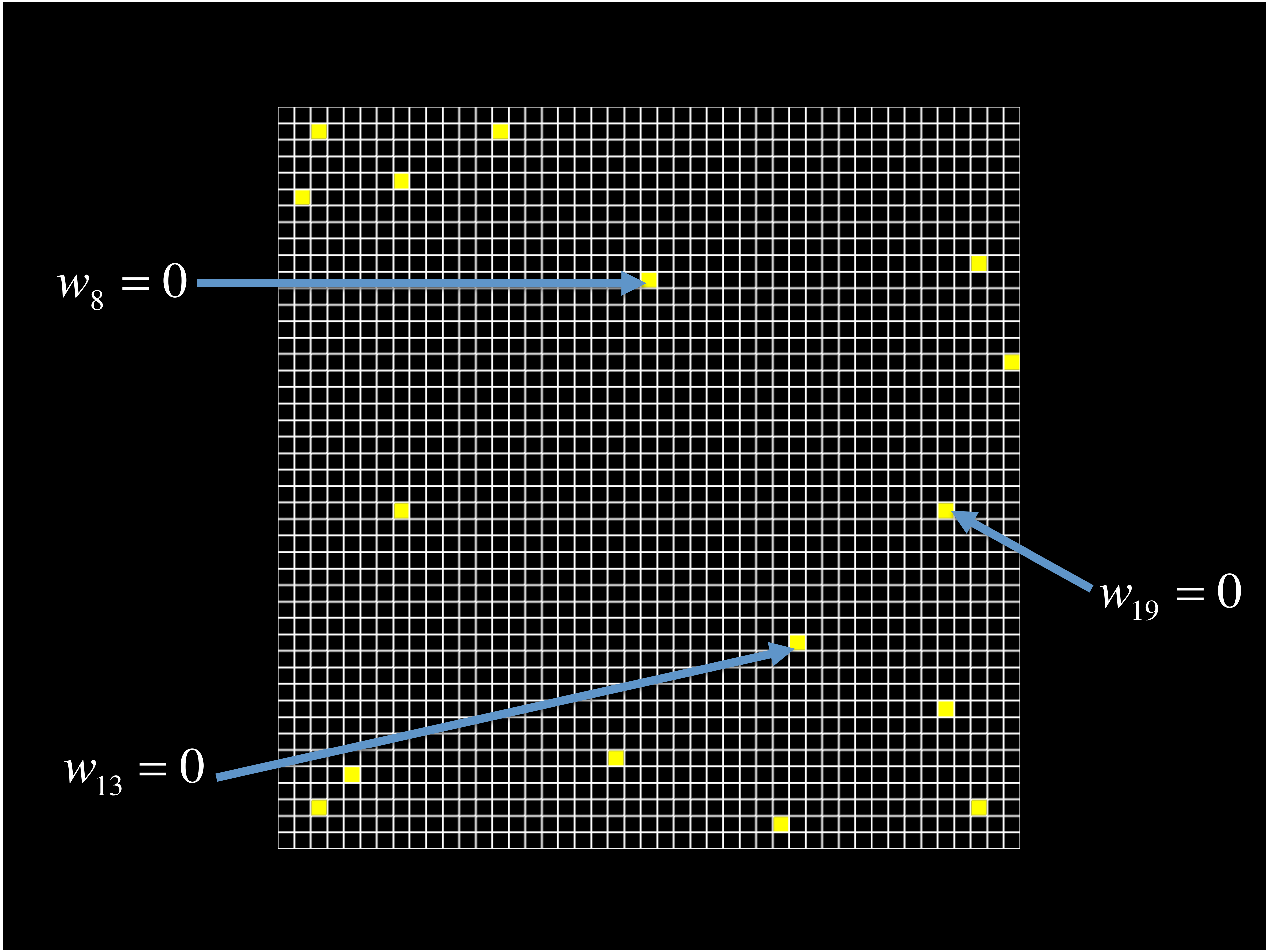


The Greedy Algorithm

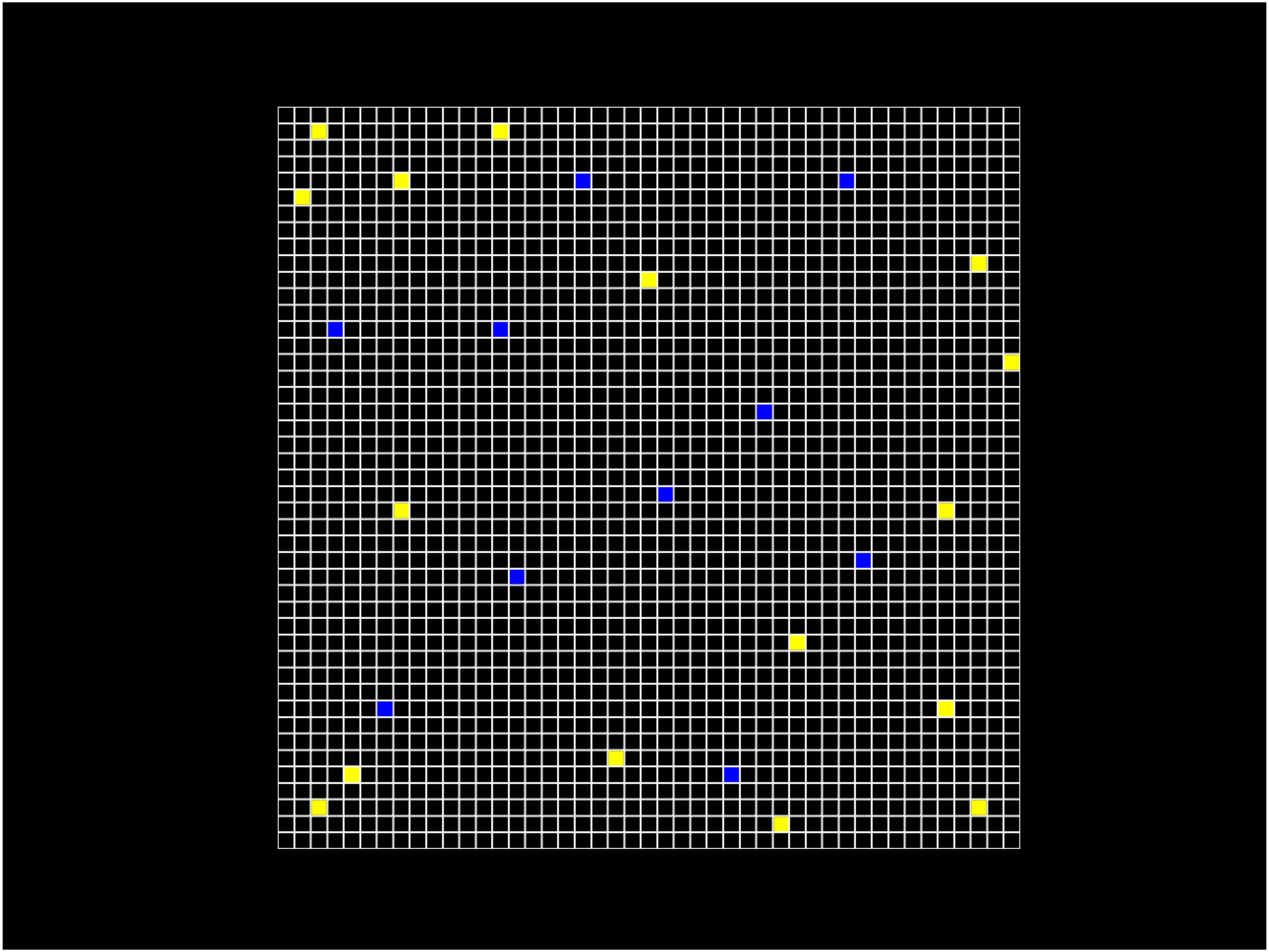


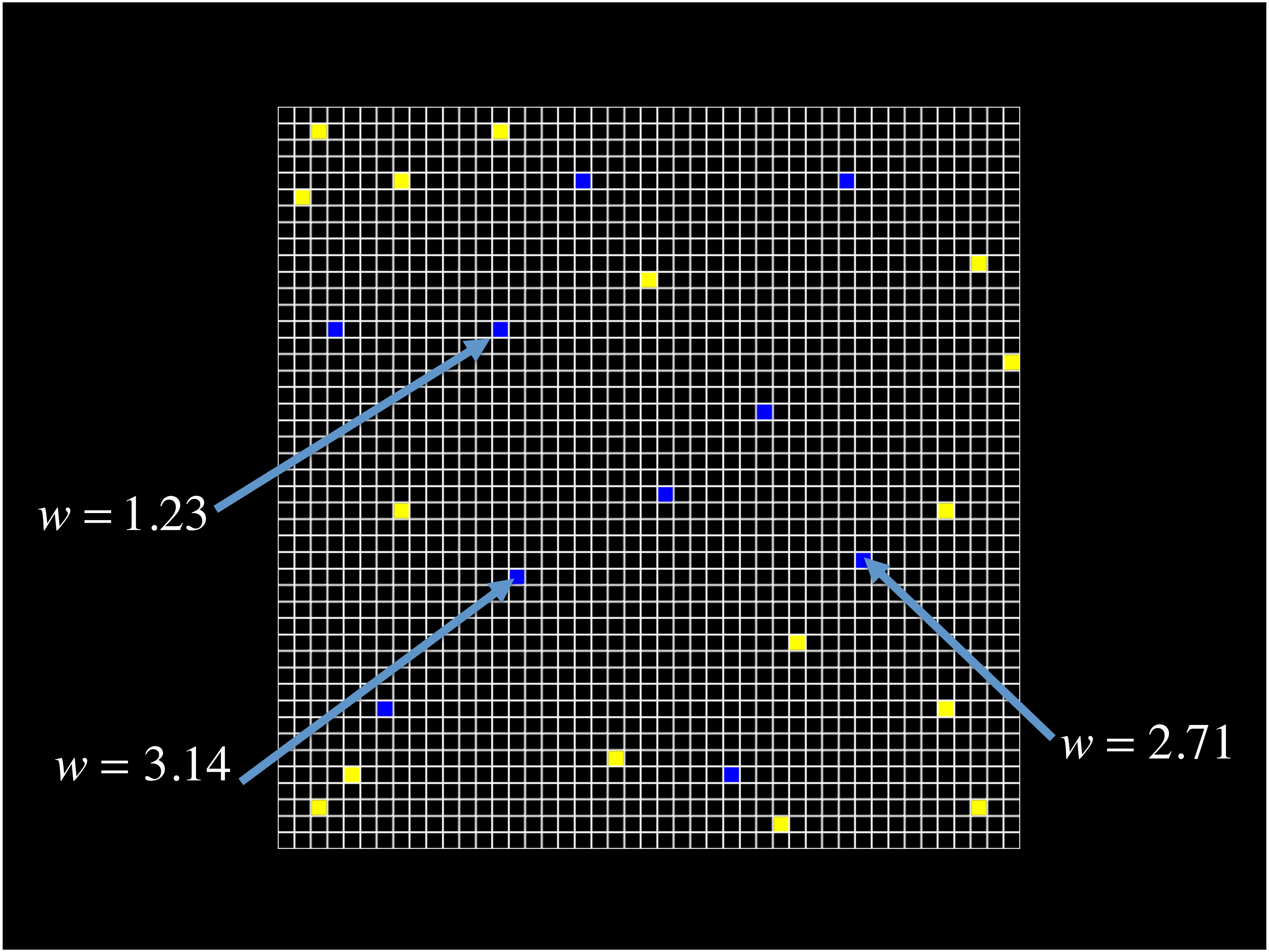
Lawson-Hanson NNLS solve: $O(P^3)$

Greedy search for P cubature points: $O(P^4)$

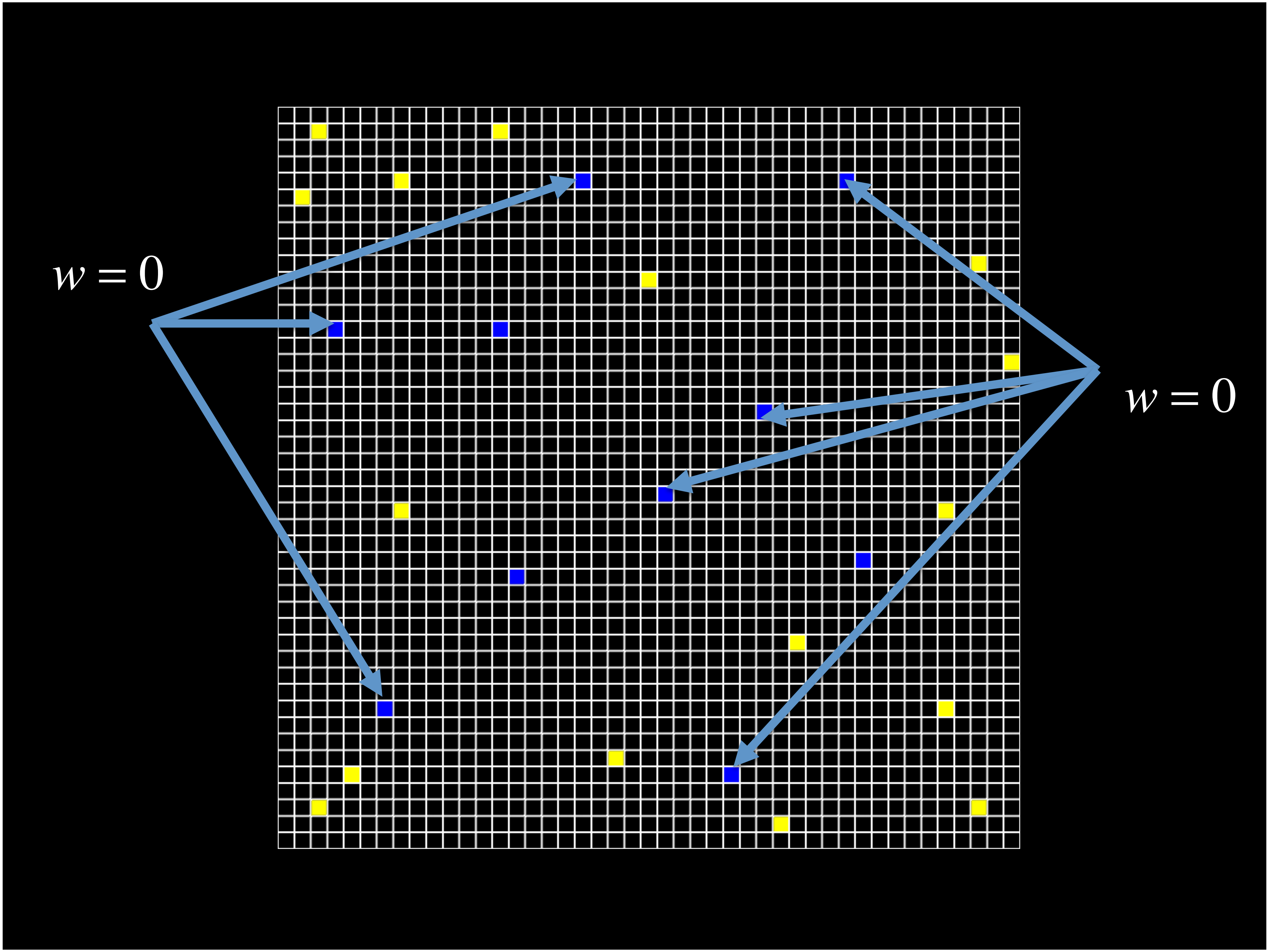


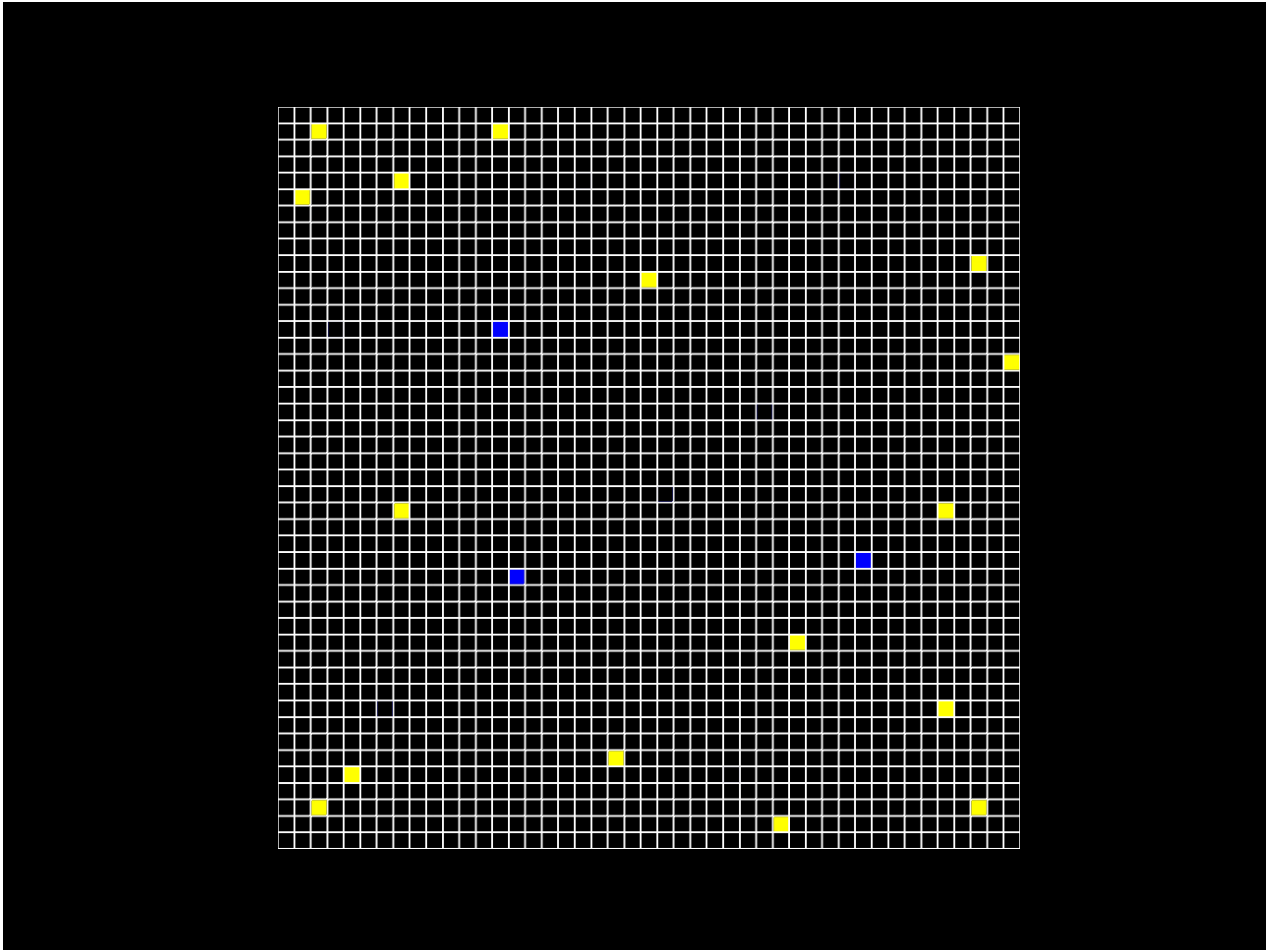
$w_8 = 0$
$w_{19} = 0$
$w_{13} = 0$





$w = 1.23$
$w = 3.14$
$w = 2.71$


$w = 0$
$w = 0$



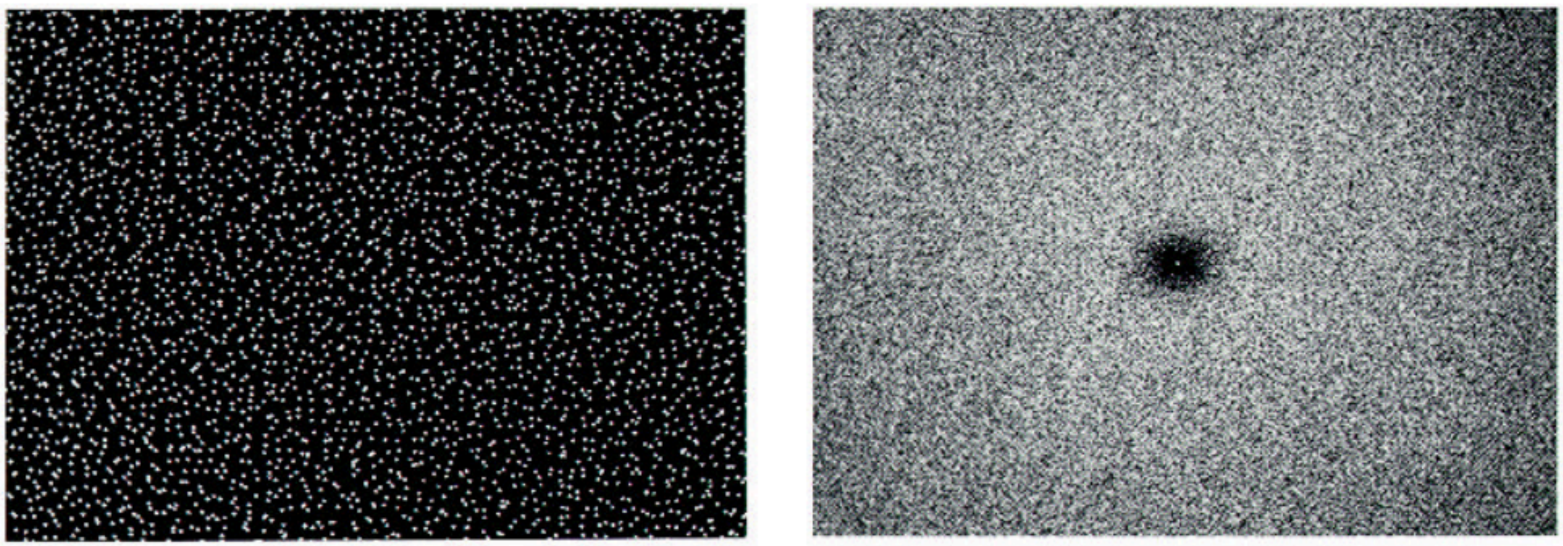
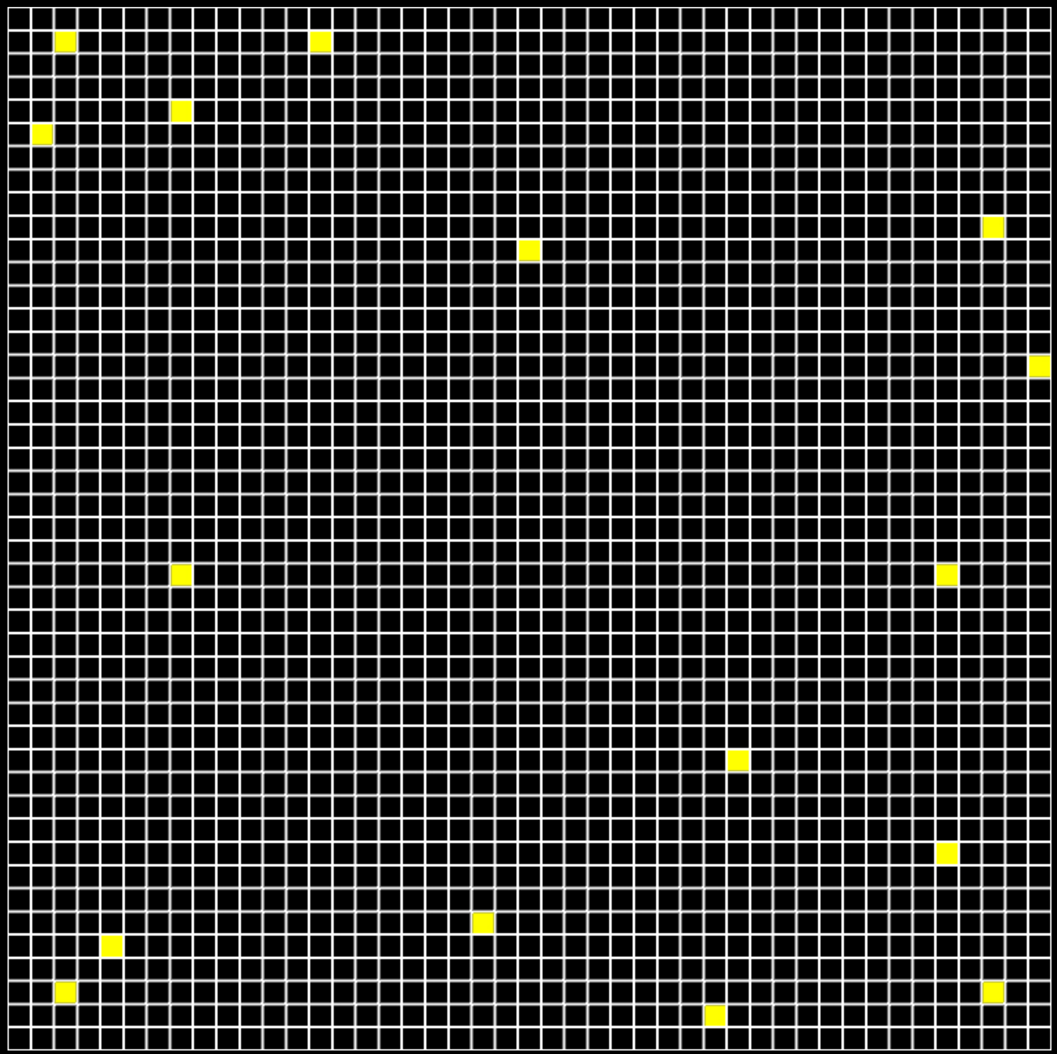


Fig. 11a. Distribution pattern of jittered samples.

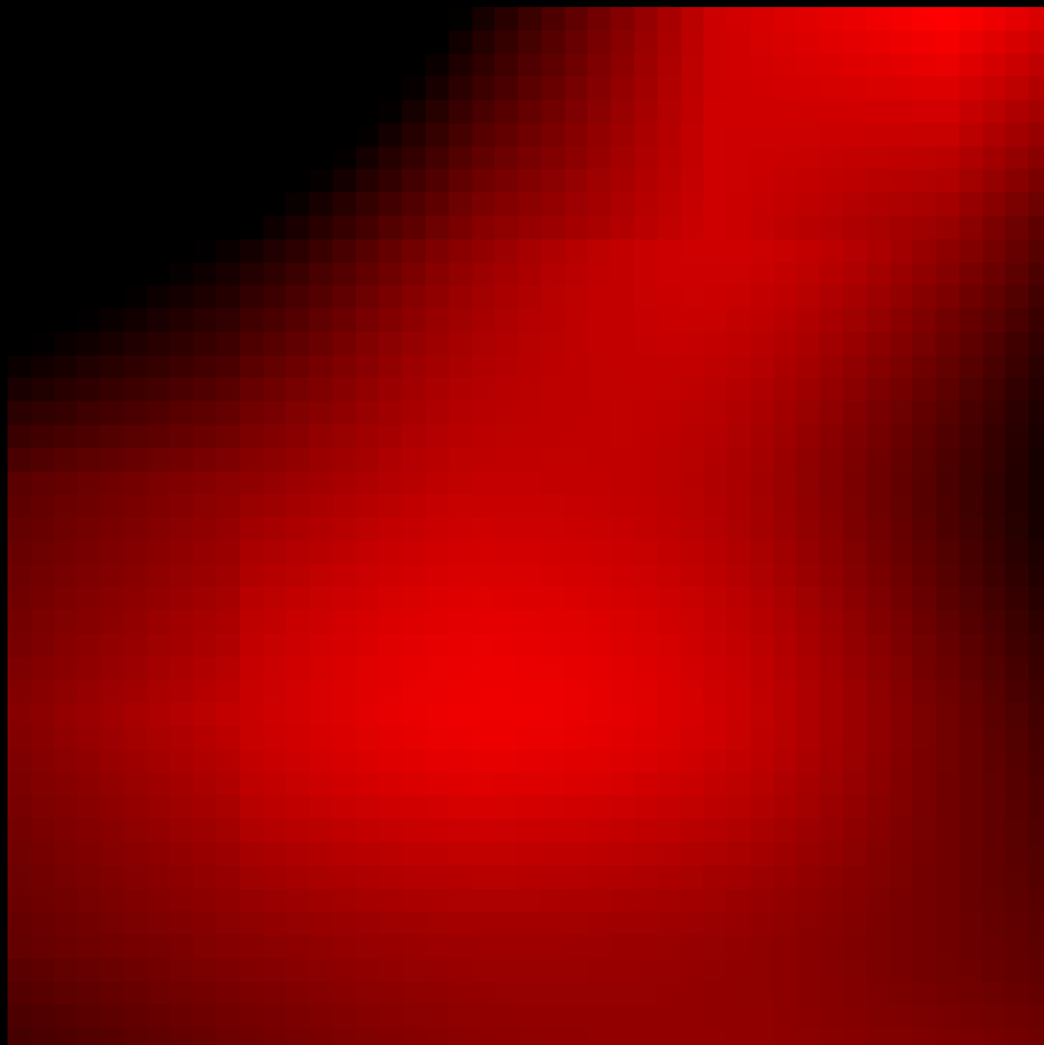
Fig. 11b. Fourier transform of the pattern in Figure 11a.

[Cook 1986]
[Pharr and Humphreys 2010]
… many others

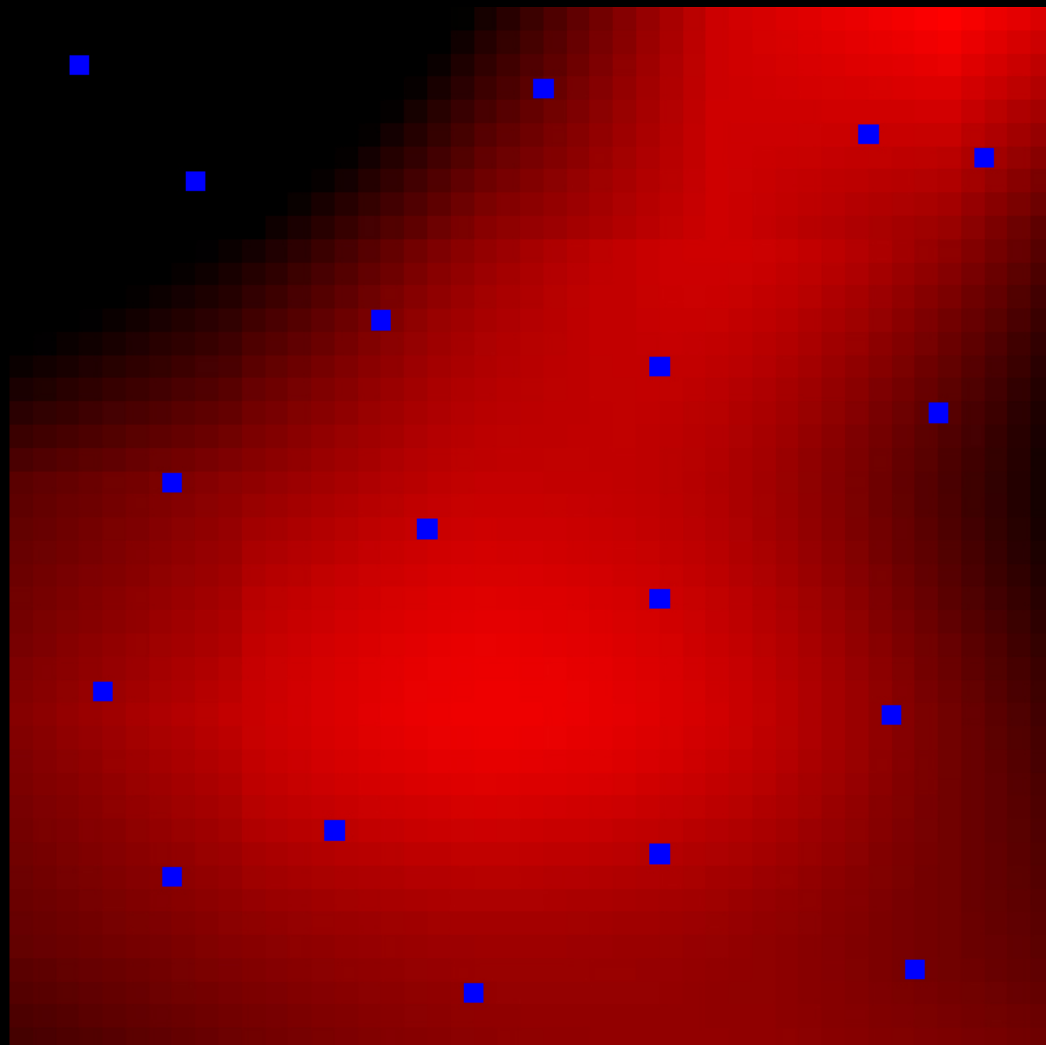
Importance Sampling



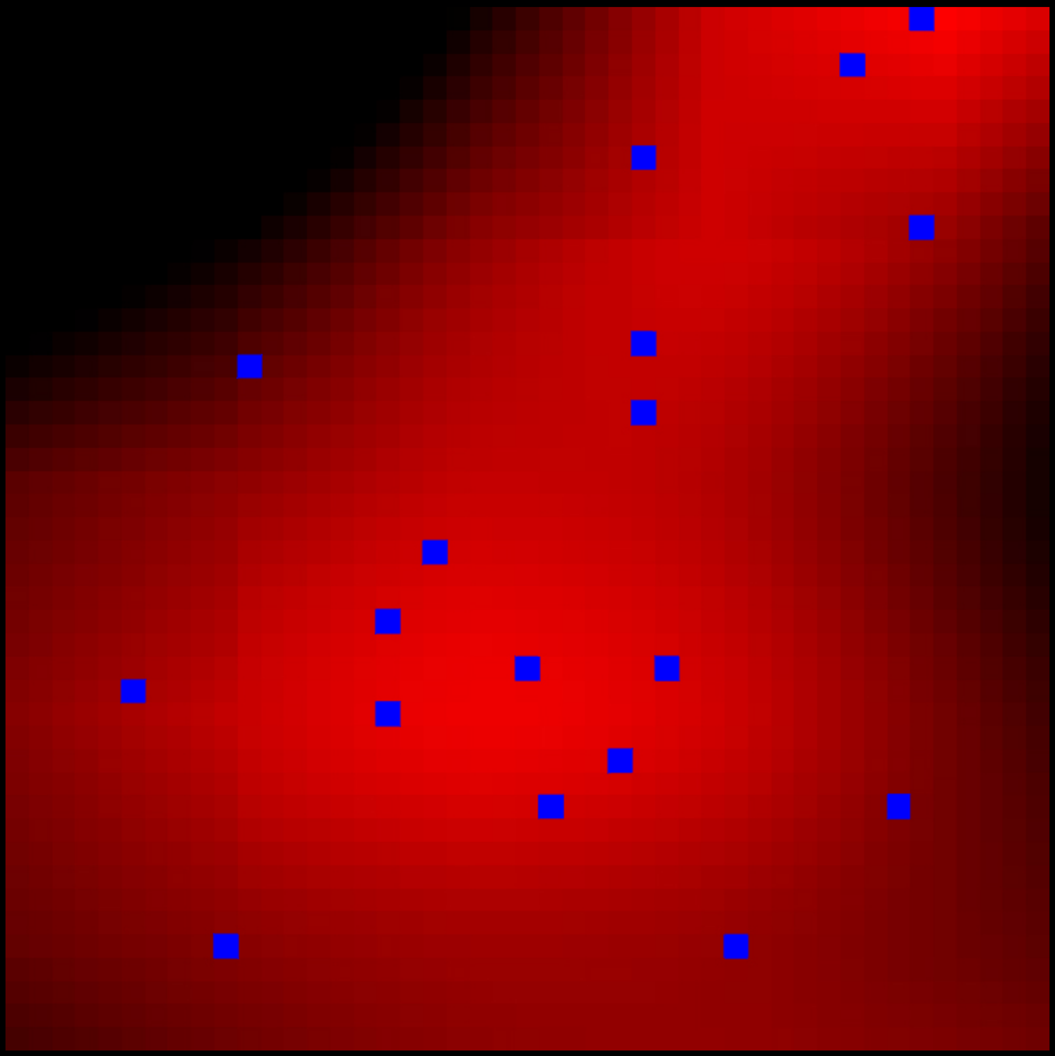
Importance Sampling



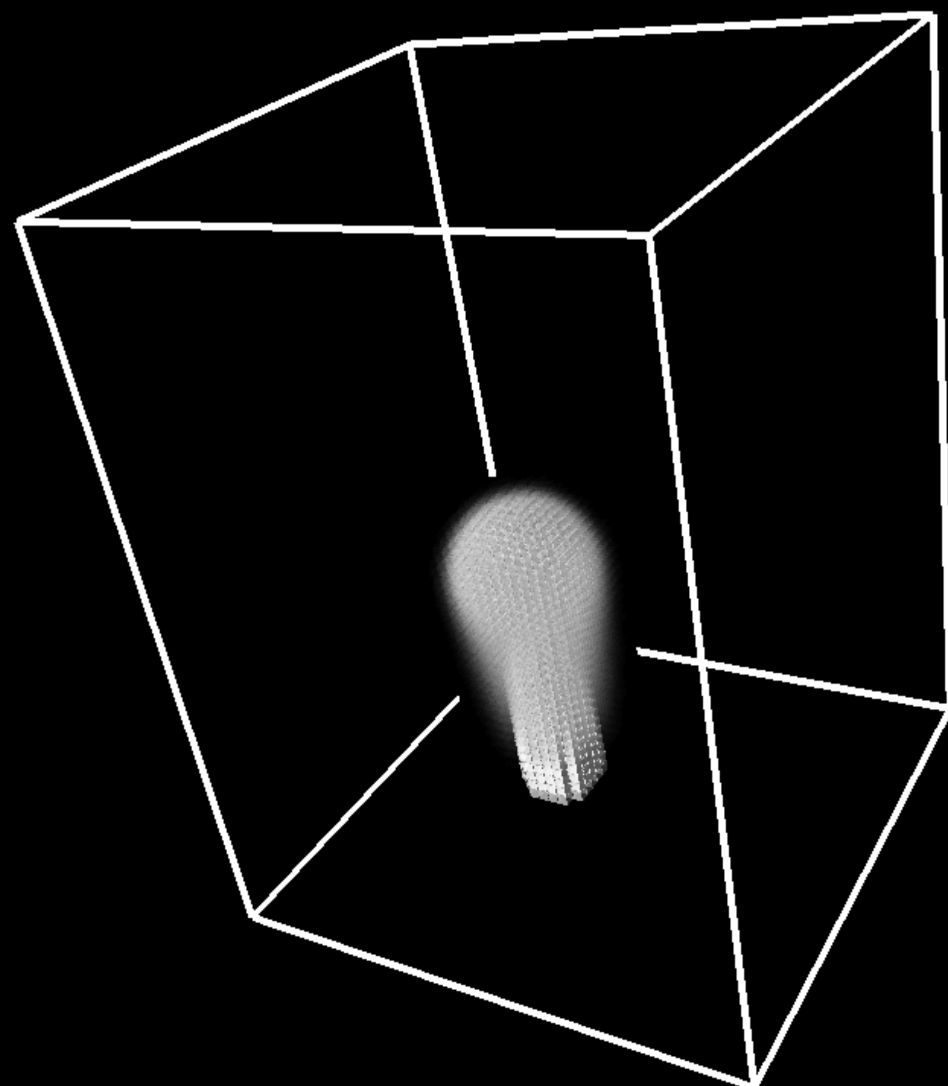
Importance Sampling



Importance Sampling



$$\mathrm{PDF}(\mathbf{x}_p) = R\left(\frac{\left|\mathbf{a}_p \cdot \mathbf{r}\right|}{\mathbf{r} \cdot \mathbf{r}}\right)$$



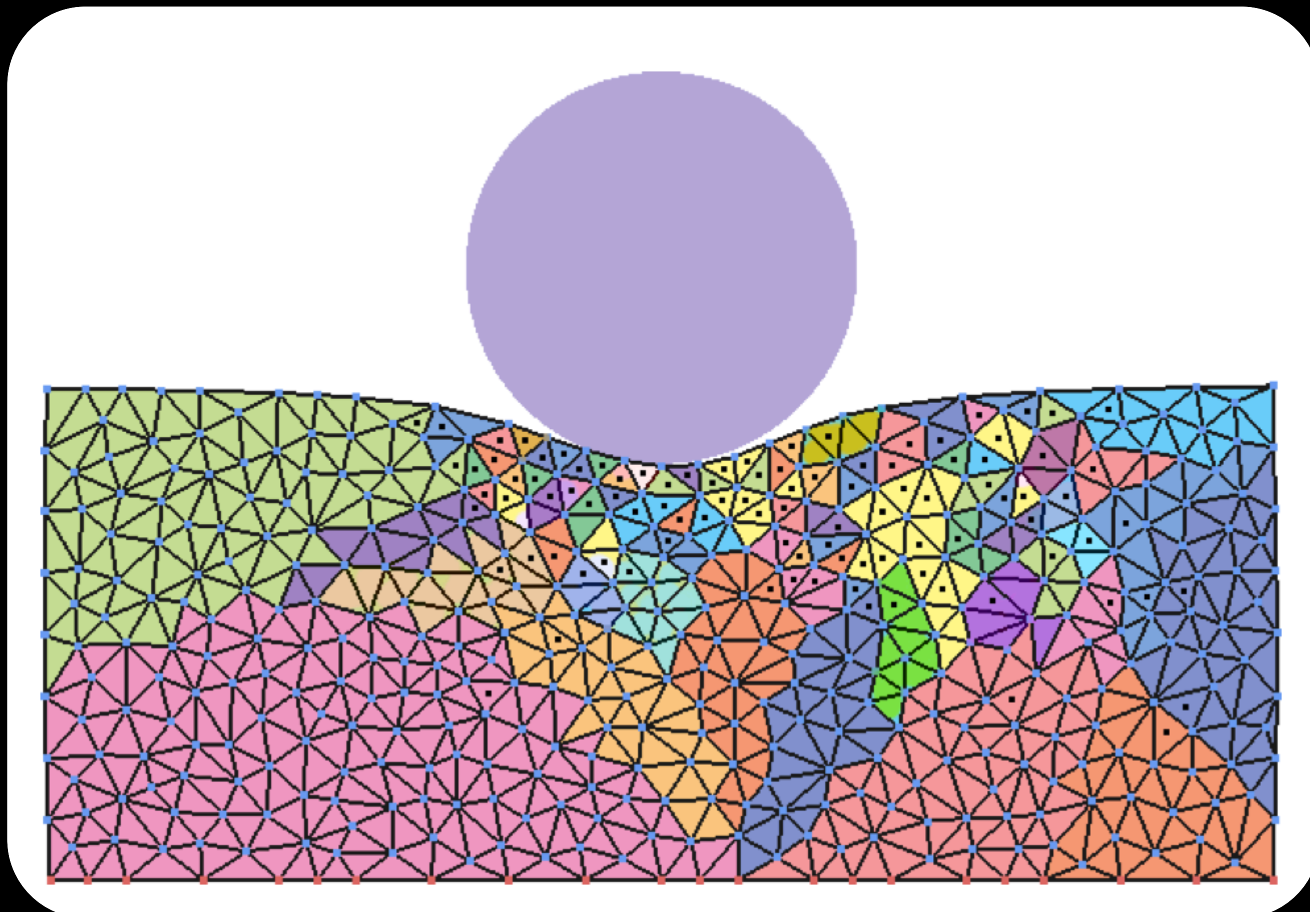
Six days ➡ 30 minutes

	Example 1	Example 2	Example 3	Example 4
L_2 error, Iteration 1	0.0428803	0.0592719	0.0409149	0.0330917
L_2 error, Iteration 2	0.0148716	0.0184463	0.0145316	0.0118481
L_2 error, Iteration 3	0.0107379	0.0112989	0.0106133	0.00650847
L_2 error, Iteration 4	0.00866083	0.00865156	0.00871744	converged
Total Time	01h 18m 07s	03h 05m 58s	09h 28m 29s	05h 29m 02s

Greedy search for P cubature points: $O(P^4)$

Importance sampled cubature: $O(P^3)$

[Harmon and Zorin 2013]



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Fast Diffusion-Projection

$$\tilde{\mathbf{u}}_2 = \tilde{\mathbf{D}}\tilde{\mathbf{u}}_1$$

$$\tilde{\mathbf{d}} = \tilde{\mathbf{W}}\tilde{\mathbf{u}}_2$$

$$\tilde{\mathbf{p}} = \tilde{\mathbf{X}}^{-1}\tilde{\mathbf{d}}$$

$$\tilde{\mathbf{u}}_{t+1} = \tilde{\mathbf{u}}_2 + \tilde{\mathbf{Y}}\tilde{\mathbf{p}}$$

Fast Diffusion-Projection

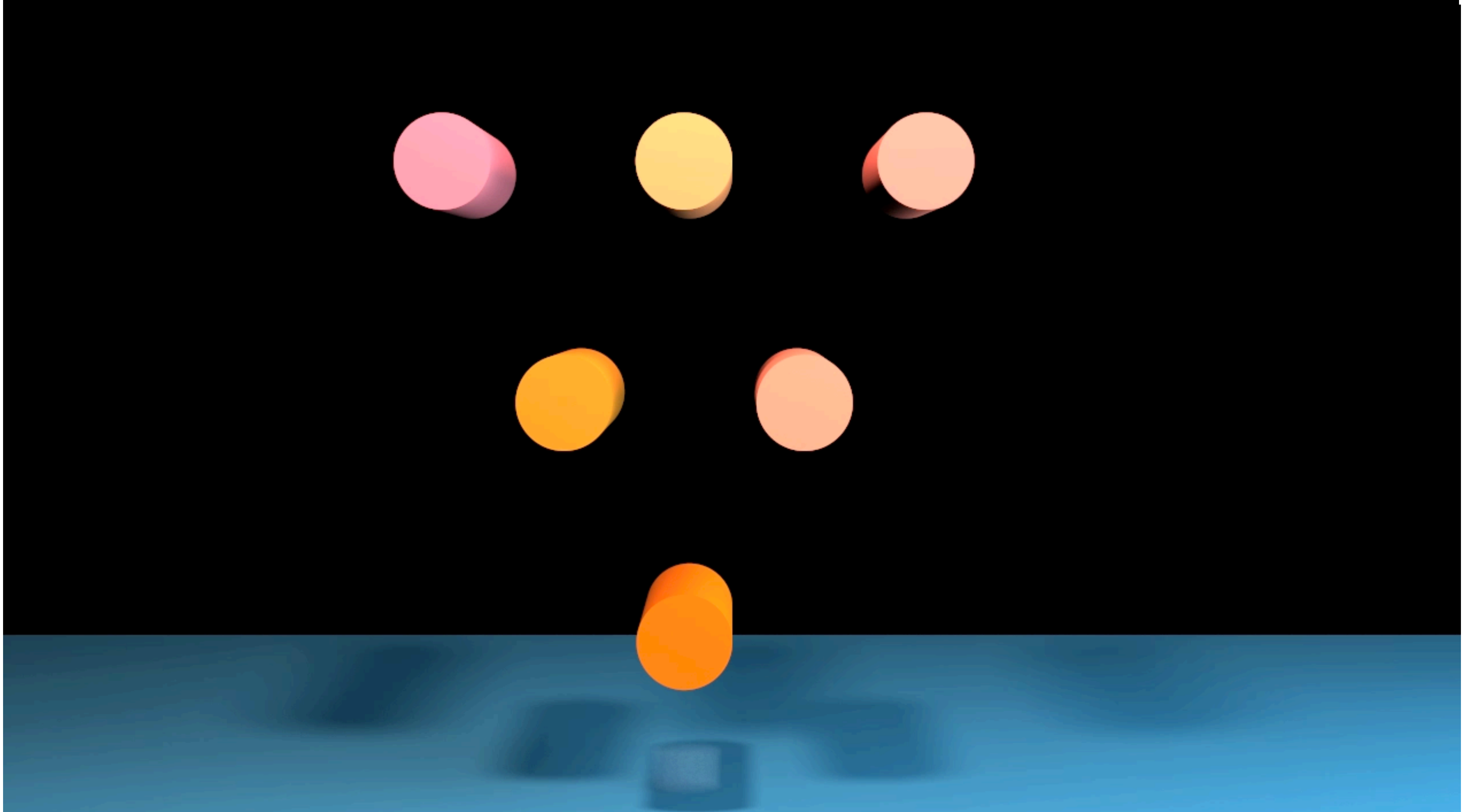
$$\left.\tilde{\mathbf{u}}_2 = \tilde{\mathbf{D}}\tilde{\mathbf{u}}_1\right\}\text{Diffusion}$$

$$\left.\begin{aligned} \tilde{\mathbf{d}} &= \tilde{\mathbf{W}}\tilde{\mathbf{u}}_2 \\ \tilde{\mathbf{p}} &= \tilde{\mathbf{X}}^{-1}\tilde{\mathbf{d}} \\ \tilde{\mathbf{u}}_{t+1} &= \tilde{\mathbf{u}}_2 + \tilde{\mathbf{Y}}\tilde{\mathbf{p}} \end{aligned}\right\}\text{Projection}$$

Fast Diffusion-Projection

$$\left.\begin{aligned}
\tilde{\mathbf{u}}_2 &= \tilde{\mathbf{D}}\tilde{\mathbf{u}}_1 \\
\tilde{\mathbf{d}} &= \tilde{\mathbf{W}}\tilde{\mathbf{u}}_2 \\
\tilde{\mathbf{p}} &= \tilde{\mathbf{X}}^{-1}\tilde{\mathbf{d}} \\
\tilde{\mathbf{u}}_{t+1} &= \tilde{\mathbf{u}}_2 + \tilde{\mathbf{Y}}\tilde{\mathbf{p}}
\end{aligned}\right\} \quad \tilde{\mathbf{u}}_{t+1} = \tilde{\mathbf{Z}}\tilde{\mathbf{u}}_1$$

Internal Obstacles



Iterated Orthogonal Projection

[Molemaker et al. 2008]

Iterated Orthogonal Projection

[Molemaker et al. 2008]

$$\tilde{\mathbf{u}}_{t+1} = \tilde{\mathbf{Z}}\tilde{\mathbf{u}}_1$$

Iterated Orthogonal Projection

[Molemaker et al. 2008]

$$\tilde{\mathbf{u}}_{t+1} = \tilde{\mathbf{Z}}\tilde{\mathbf{u}}_1$$

$$\tilde{\mathbf{u}}_{t+1} = \tilde{\mathbf{Z}}\tilde{\mathbf{P}}\tilde{\mathbf{u}}_1$$

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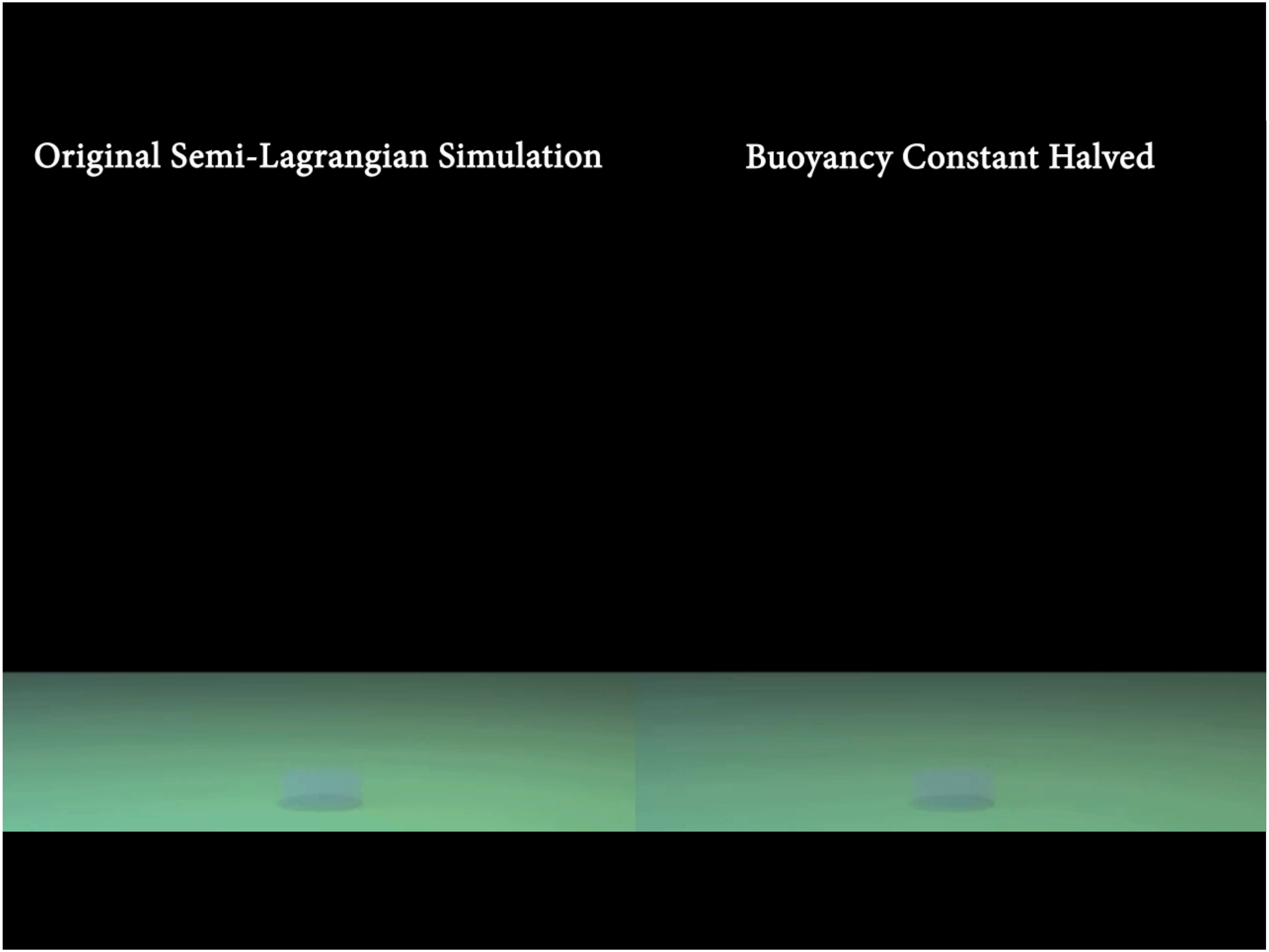
Stam Plume, 200×266×200
06h 57m 30s

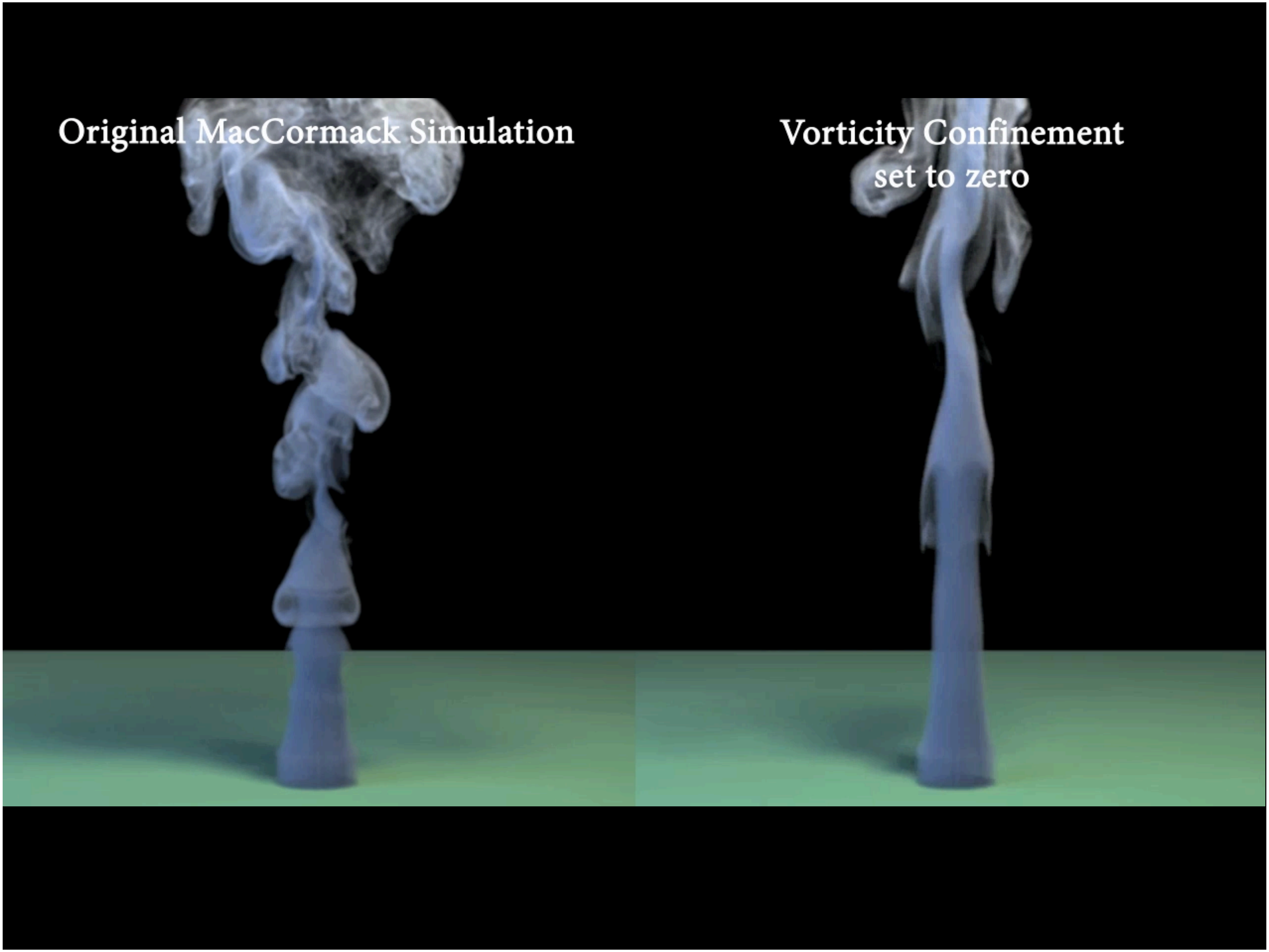

Original Semi-Lagrangian Simulation
Buoyancy Constant Halved

Stam Plume example

Solver Only:	18ms	9326x faster
With Vel. Recon.:	4.2s	39x faster

Total preprocessing: 09h 50m 23s

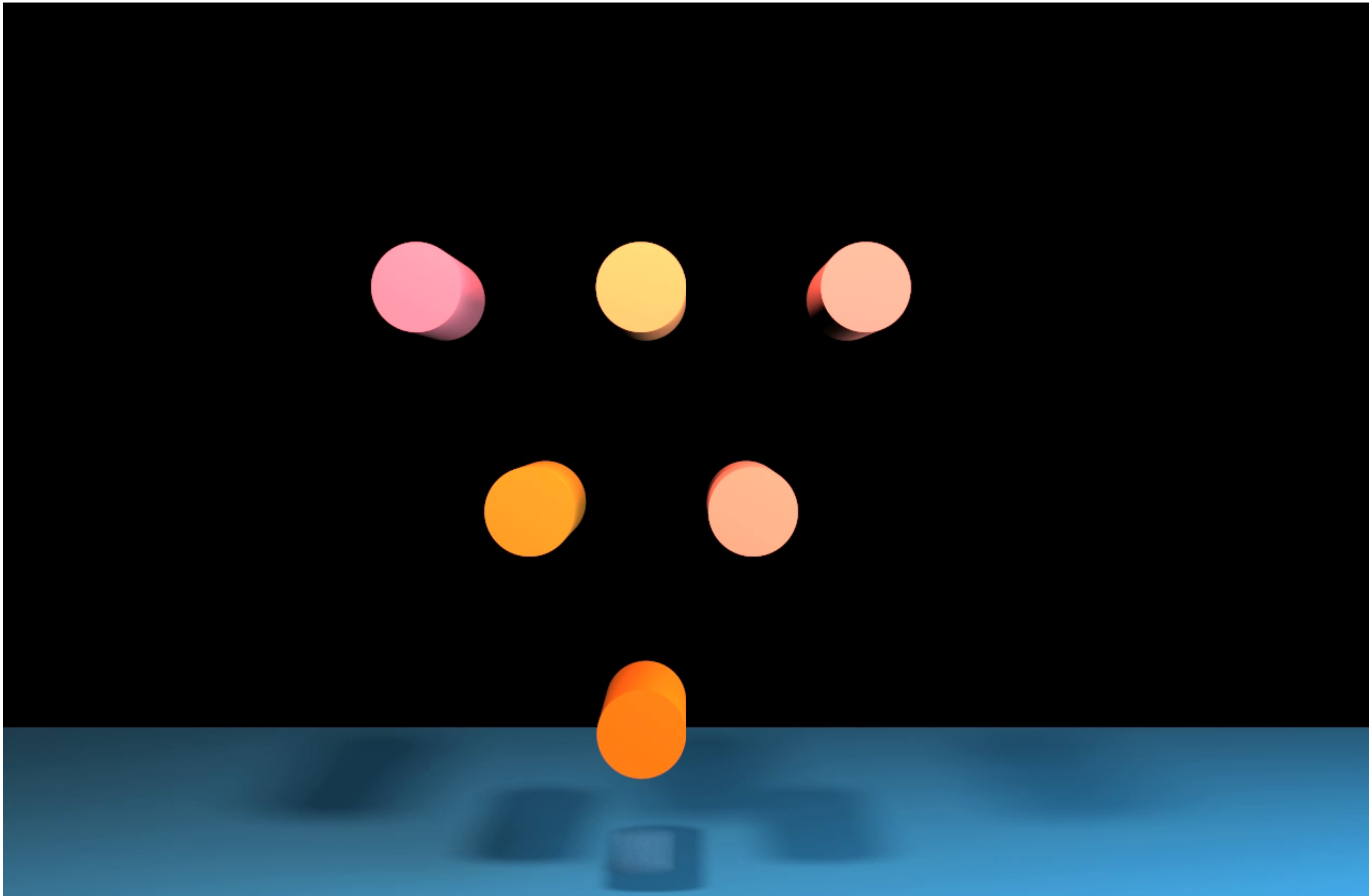
12-core, 2.66 Ghz Mac Pro


Original MacCormack Simulation
Vorticity Confinement
set to zero

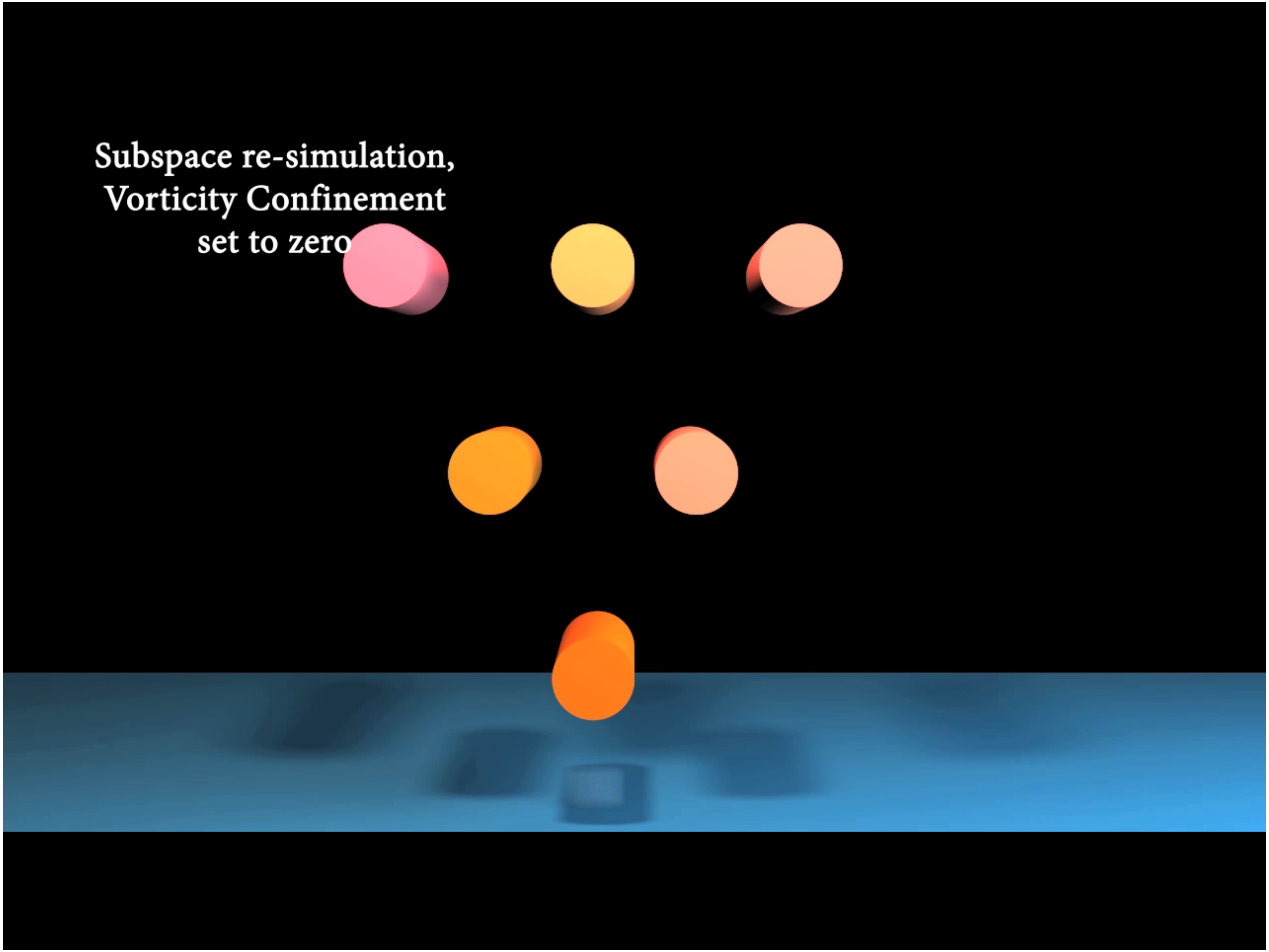
MacCormack example

Solver Only:	96ms	1764x faster
With Vel. Recon.:	5.6s	30x faster

Total preprocessing:	09h 27m 16s



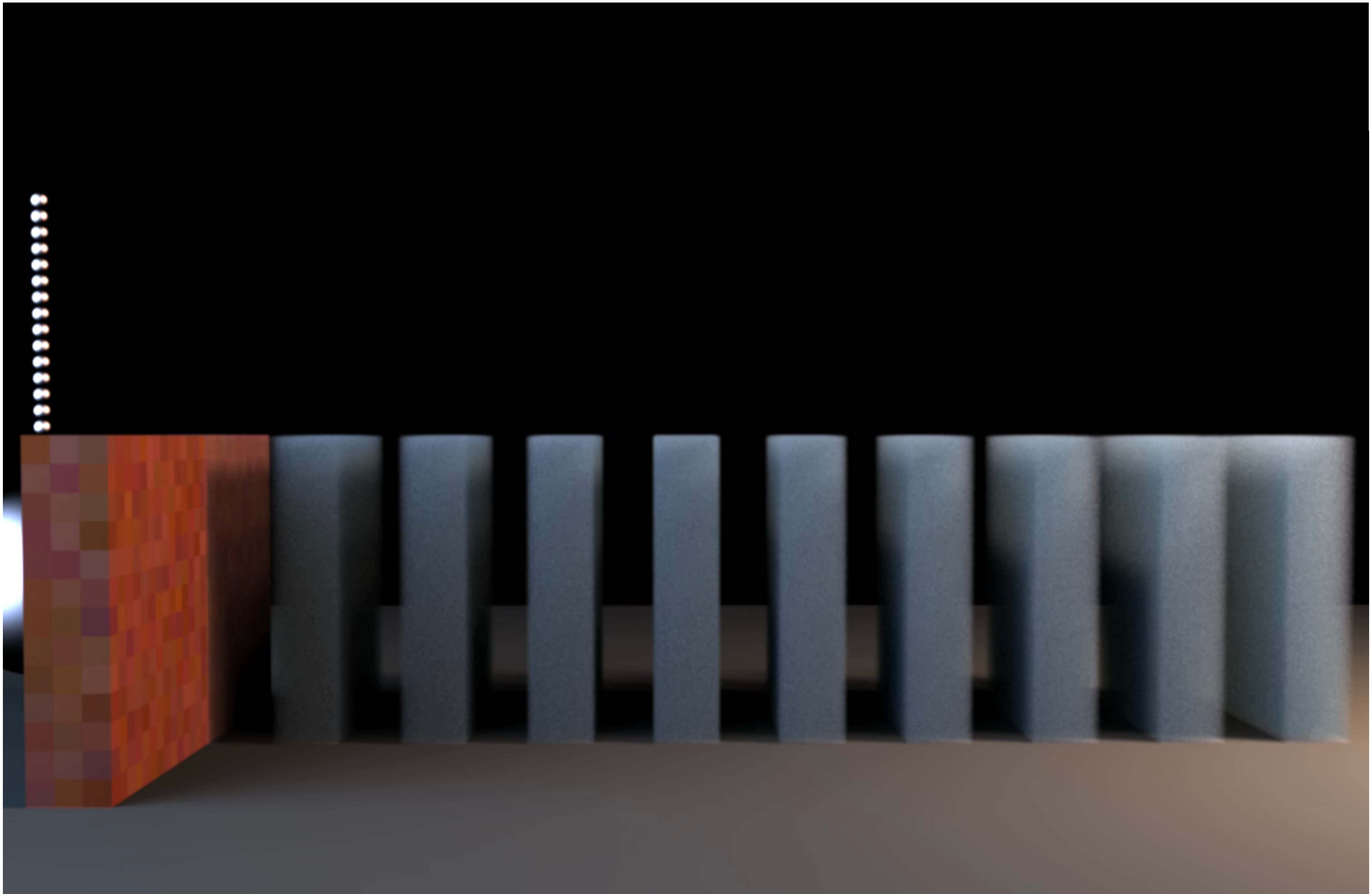
Dirichlet obstacles, 276 x 276 x 138
03h 35m 00s


Subspace re-simulation,
Vorticity Confinement
set to zero

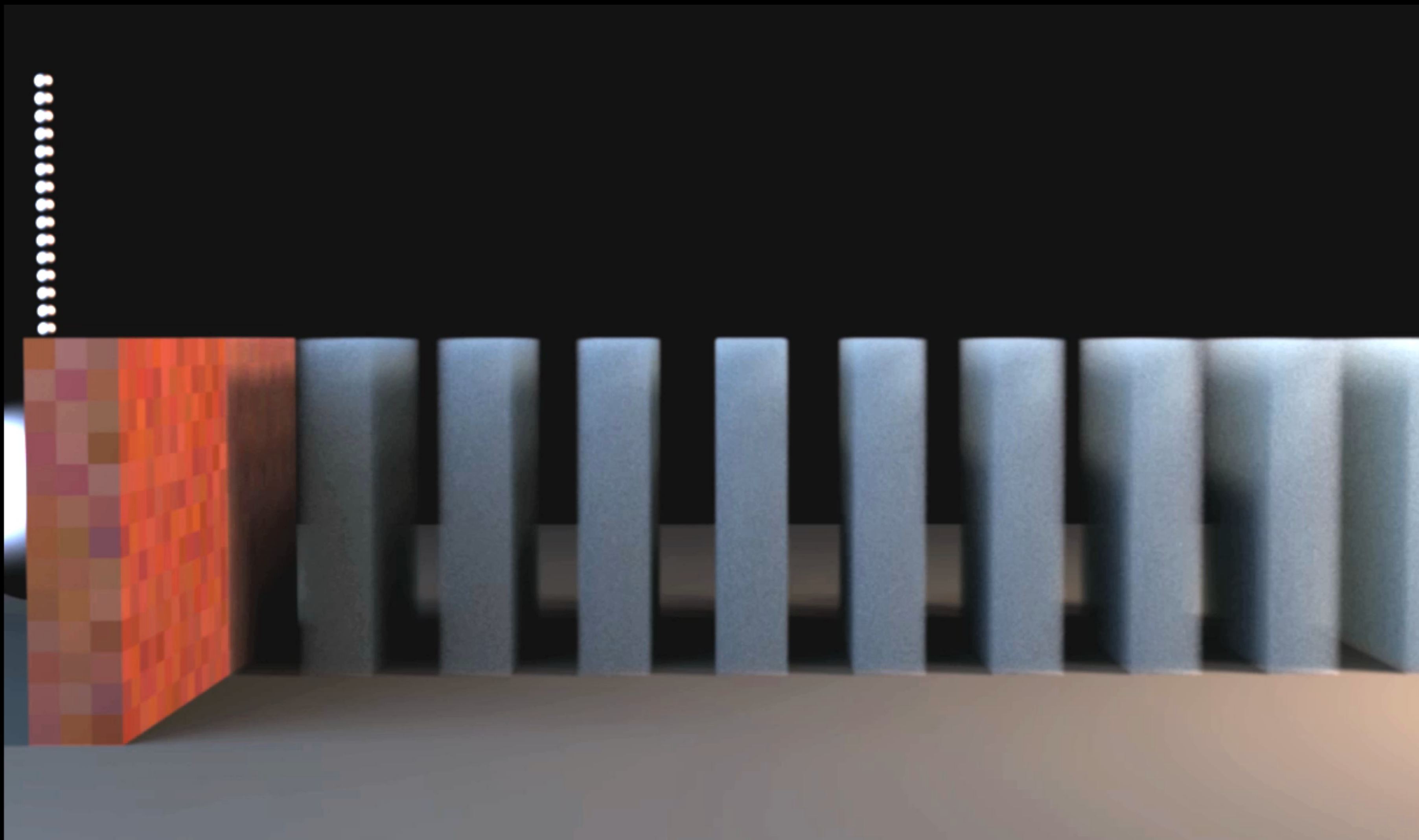
Dirichlet example

Solver Only:	130ms	661x faster
With Vel. Recon.:	5.1s	17x faster

Total preprocessing: 19h 00m 58s



Neumann obstacles, 175 x 175 x 350
03h 27m 30s



Vorticity confinement = 20, originally 1.5

Neumann example

Solver Only:	34ms	2435x faster
With Vel. Recon.:	5.7s	14x faster

Total preprocessing: 18h 53m 55s

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Contributions

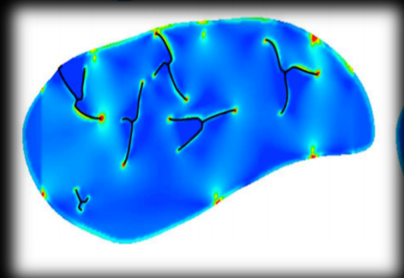
- Fast *re-simulation* of an existing simulation
- A *cubature* approach to subspace advection
- Practical cubature training via *importance sampling*
- Internal obstacles via subspace *iterated orthogonal projection*

Limitations

- Memory intensive (Mac Pro had 96 GB)
- Time-consuming pre-process
- How well does it generalize?

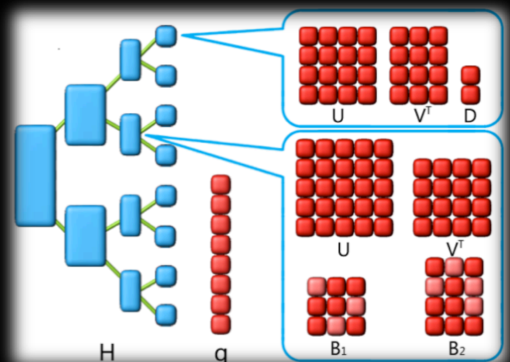
Future Work

- Basis enrichment (XFEM?)



[Richardson et al. 2011]

- Better basis compression (HSS?)



[Seo et al. 2011]

- Liquid re-simulation?

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