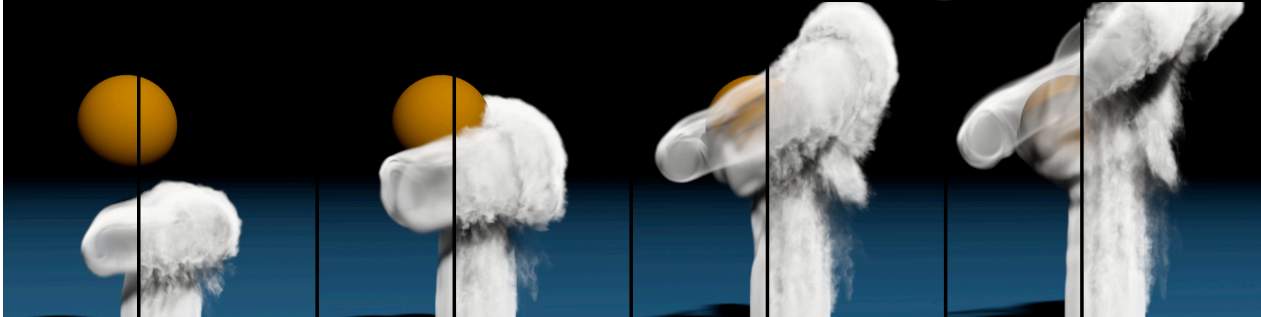


Wavelet Turbulence for Fluid Simulation



Theodore Kim¹, Nils Thürey², Doug James¹, and Markus Gross²



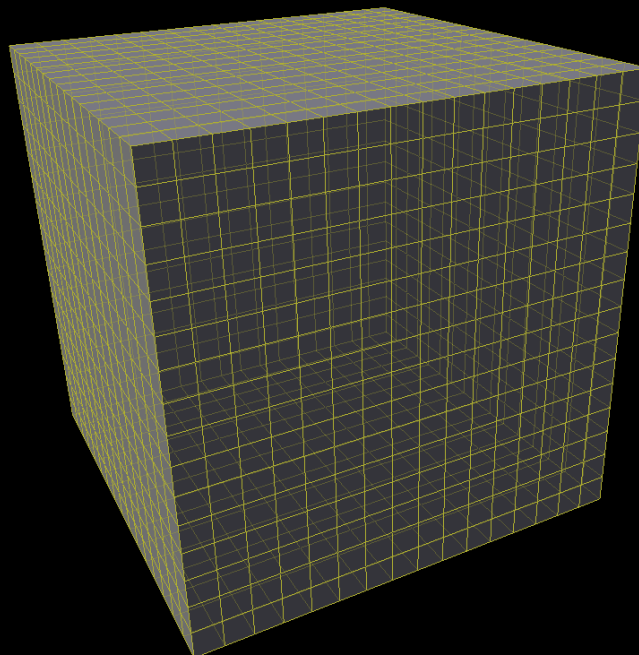
Cornell University¹

ETH²

Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

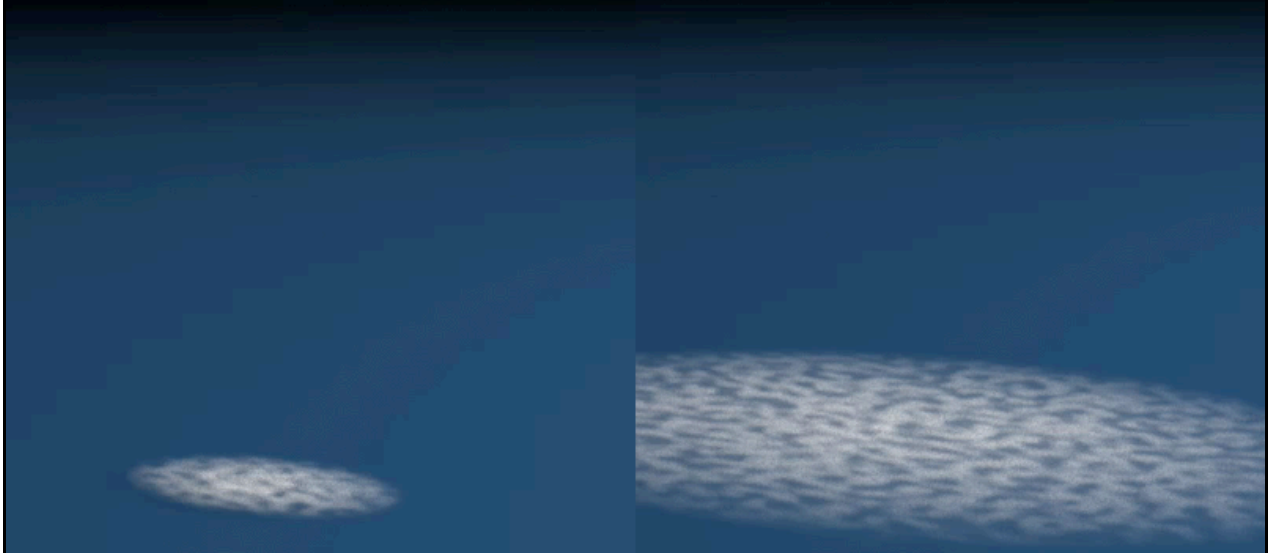
Large-Scale Fluids

- Brute force fluids are time and memory intensive



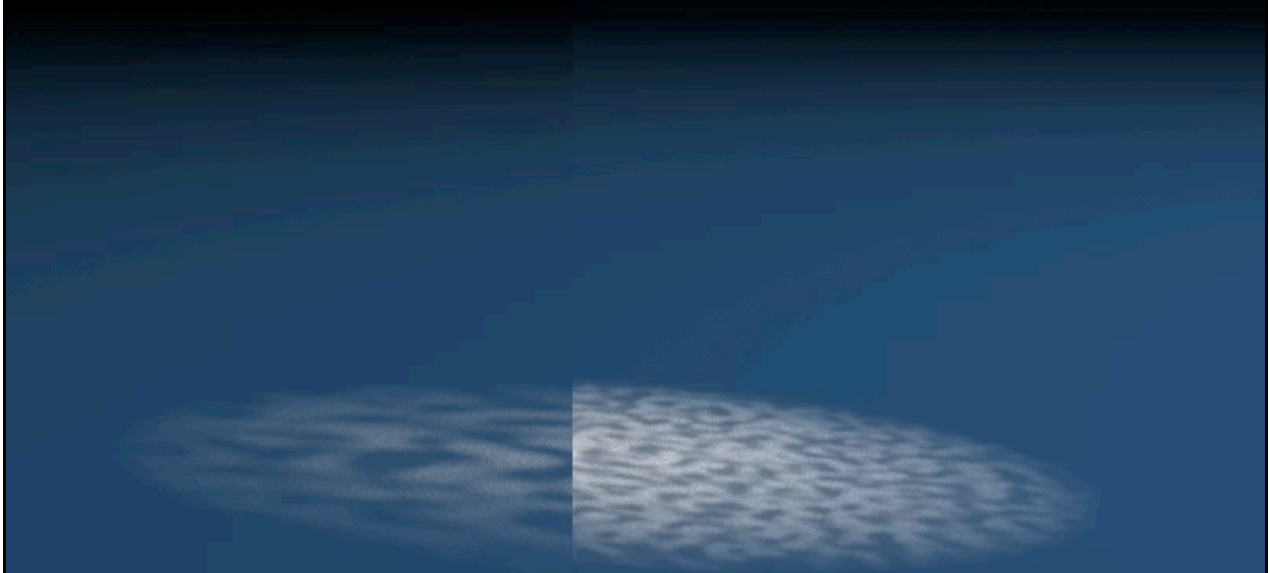
Small

Large



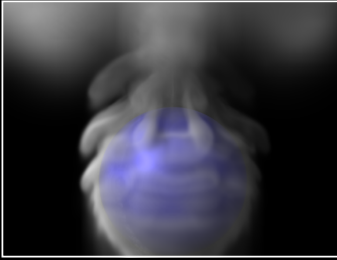
Without
Turbulence

With
Turbulence

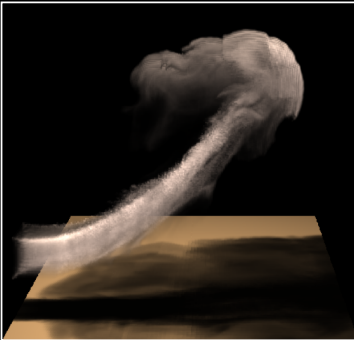


Adaptive Refinement

- Octrees

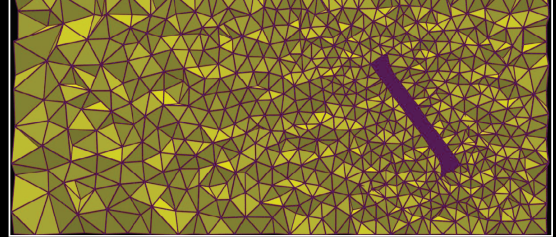
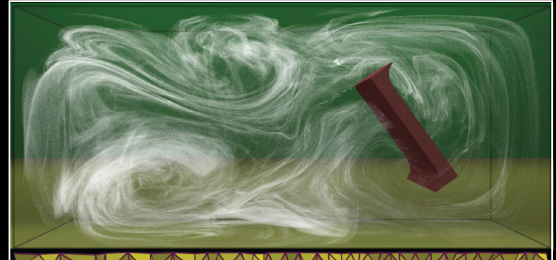


[Losasso et al. 2004]



[Shi and Yu 2004]

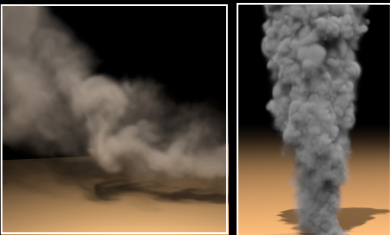
- Graded Tetrahedra



[Klingner et al. 2006]

Dissipation Suppression

- Vortex Methods



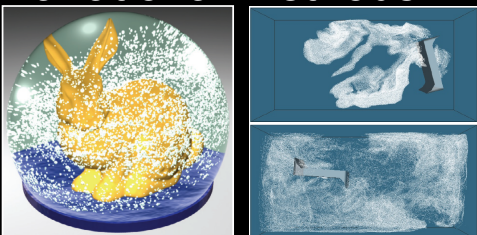
[Fedkiw et al. 2001] [Selle et al. 2005]

- QUICK



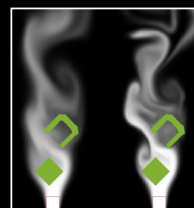
[Molemaker et al. 2008]

- Variational Methods



[Elcott et al. 2007] [Batty et al. 2007]

- MacCormack / BFECC



[Kim et al. 2006]
[Selle et al. 2008]

Turbulence Modeling

- How does fluid behave at different scales?



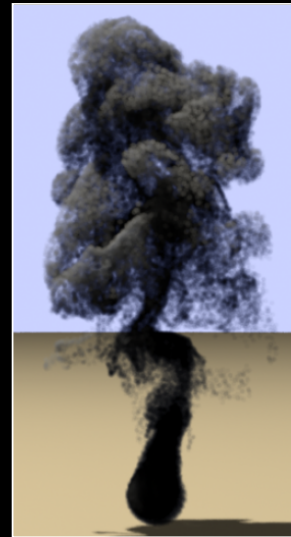
[Stam and Fiume 1993]



[Lamorlette and Foster 2002]



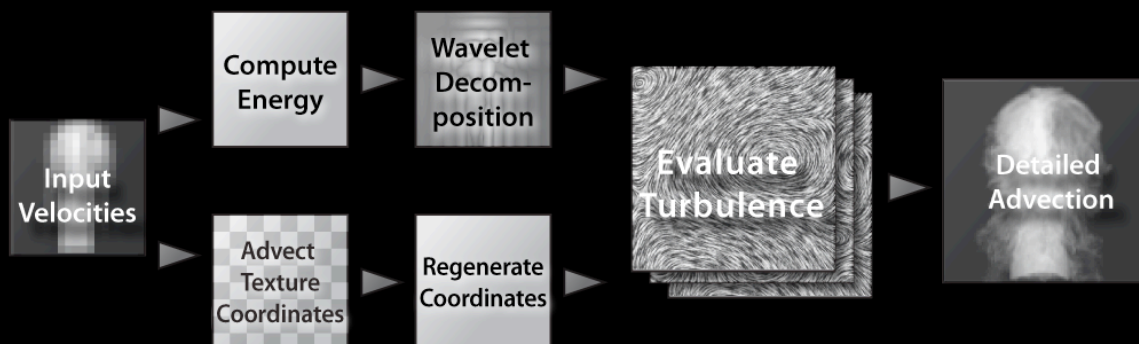
[Rasmussen et al. 2003]



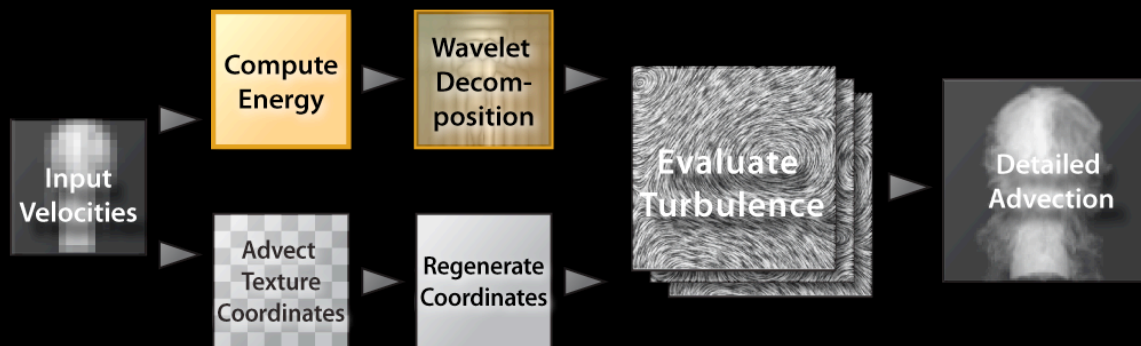
[Schechter and Bridson 2008]

[Narain et al. 2008]

Overview

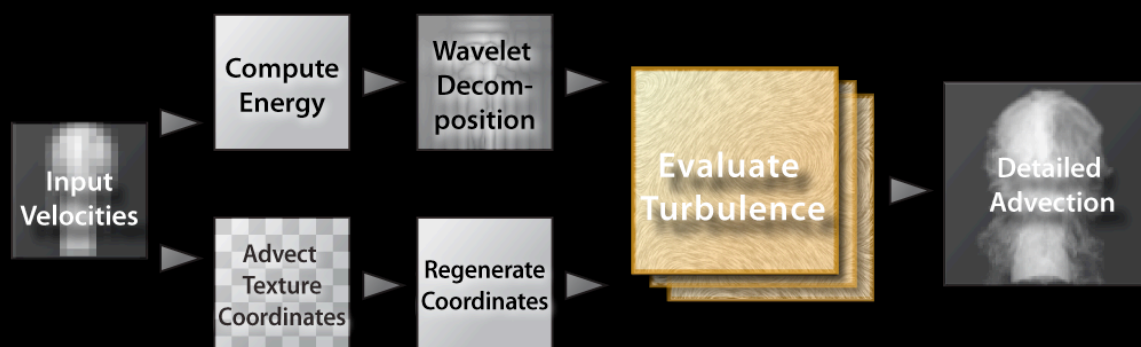


Overview



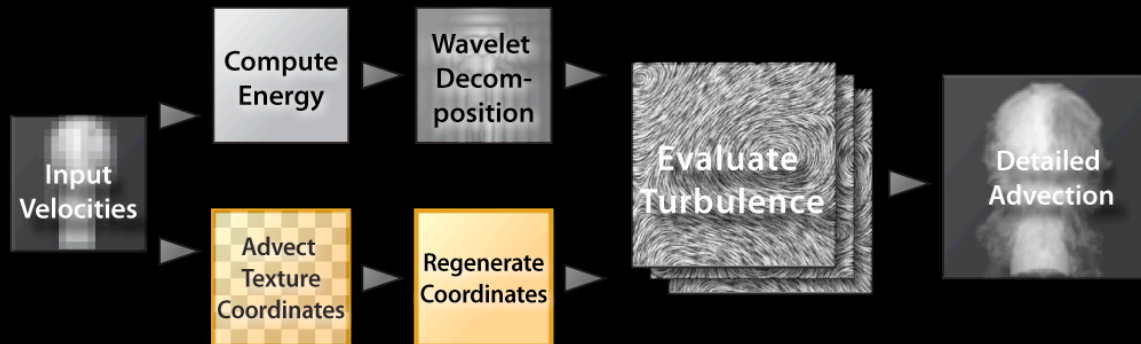
- Where are fine details lost?
 - Detect with the wavelet transform

Overview



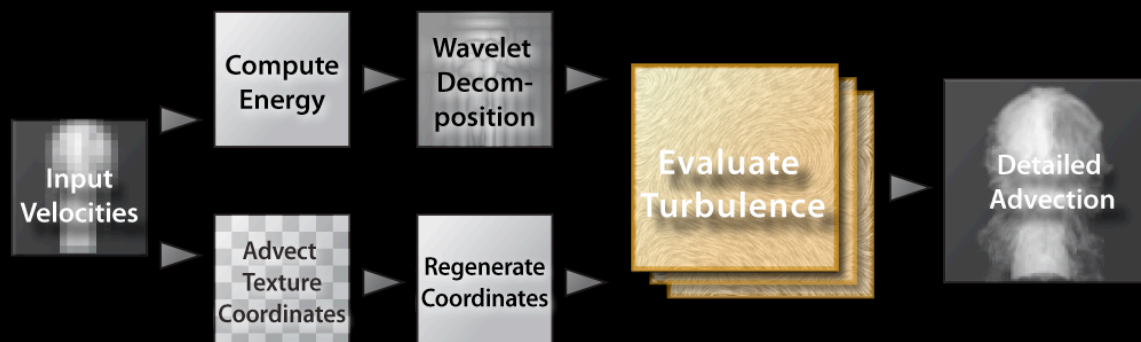
- How to efficiently add them back?
 - Modify Wavelet Noise [Cook and DeRose 2005]

Overview



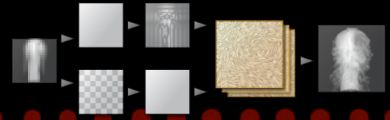
- Can we make it follow the flow?
 - Use texture advection

Overview

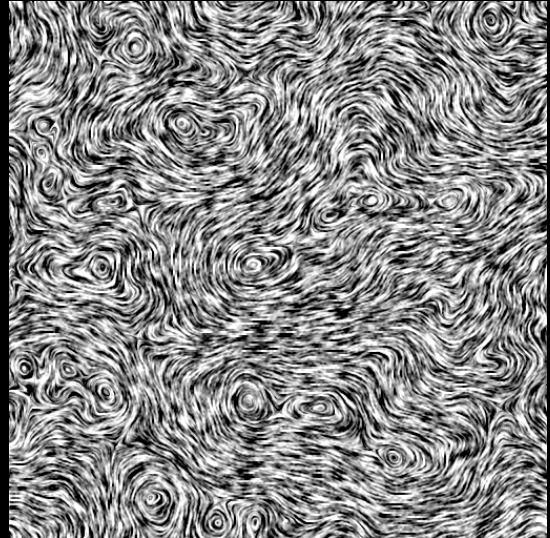
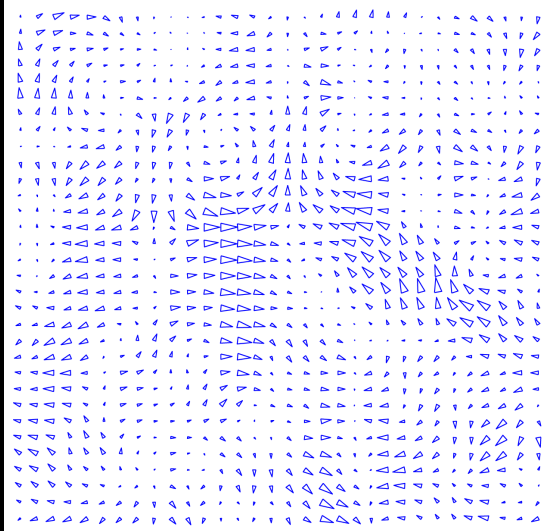


Kolmogorov Turbulence

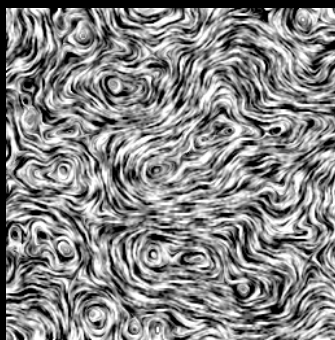
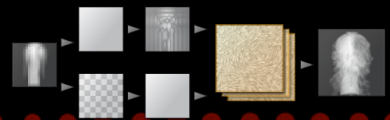
[Kolmogorov 1941]



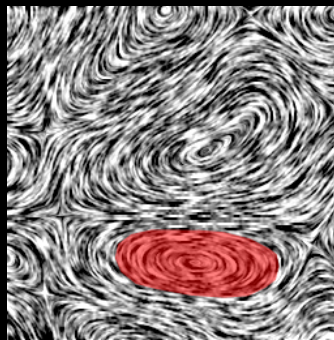
- A velocity field:



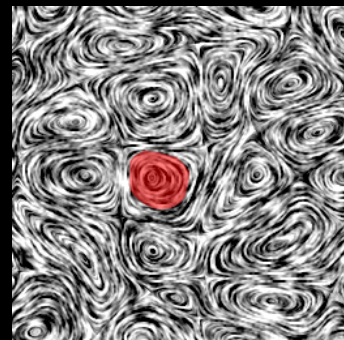
Frequency Decomposition



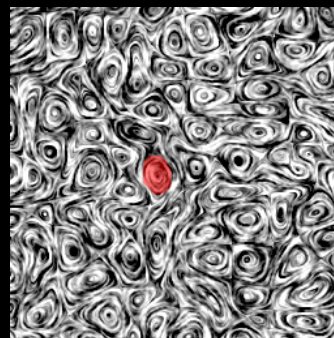
=



+



+

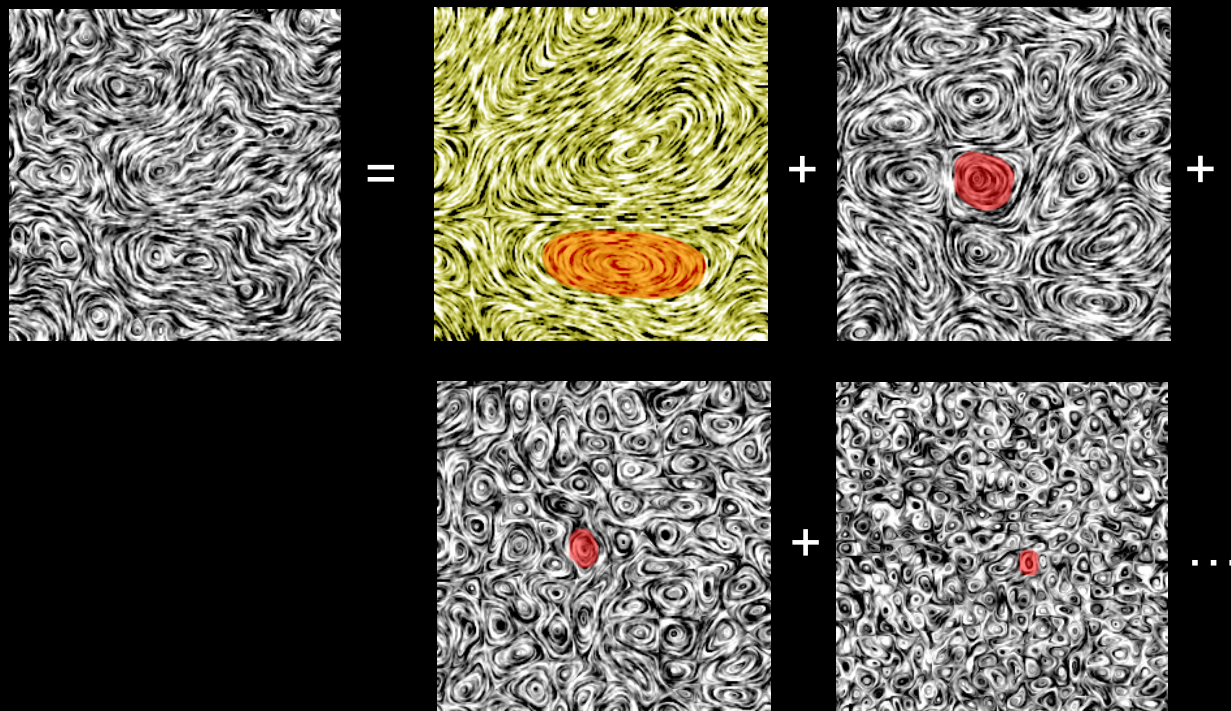


+

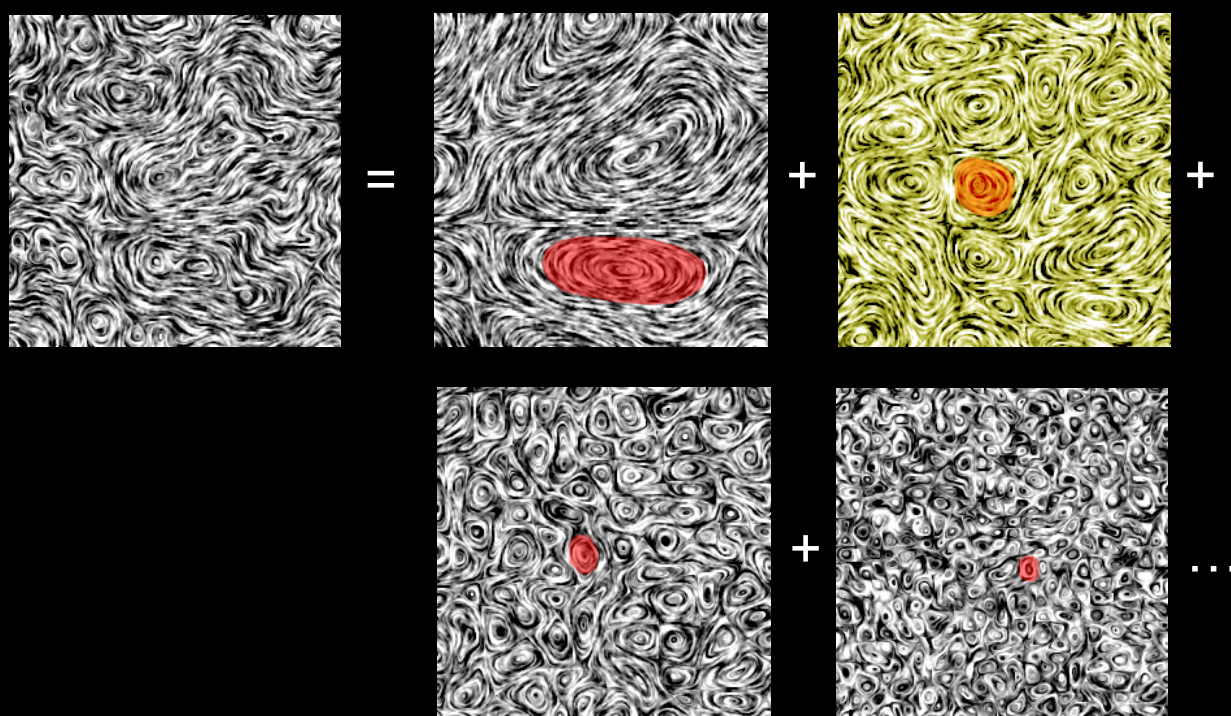
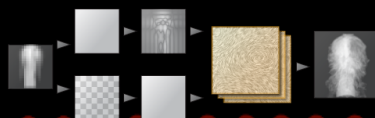


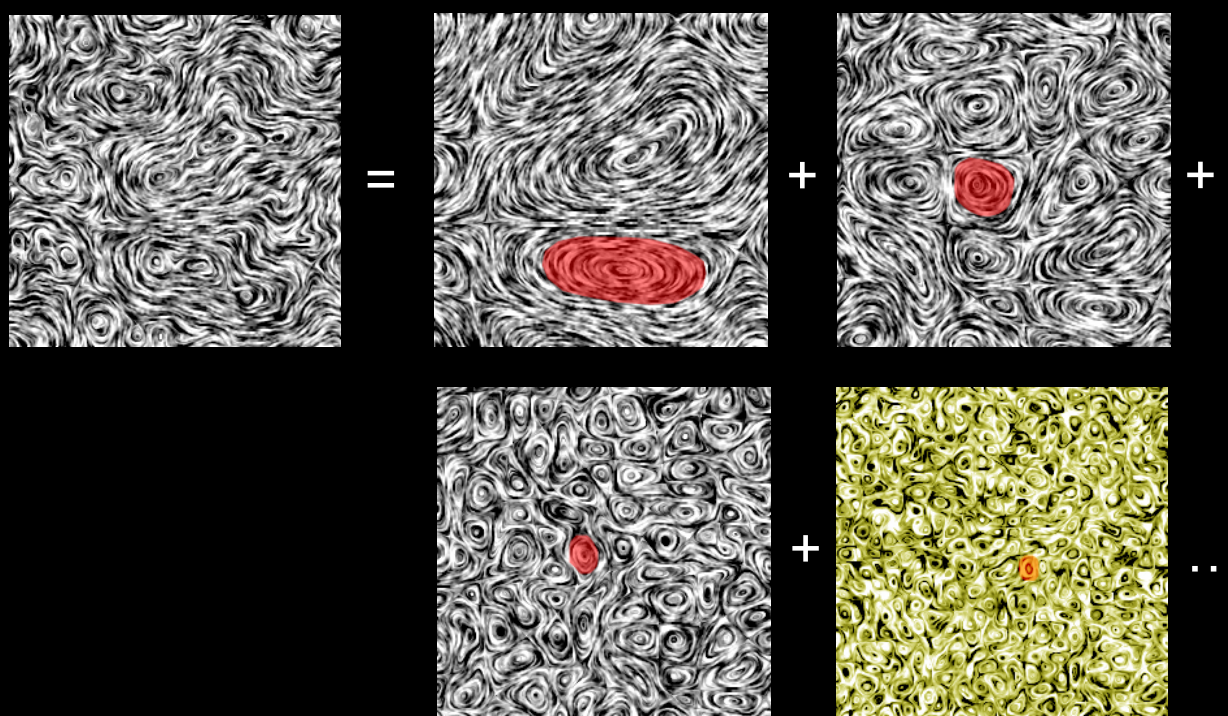
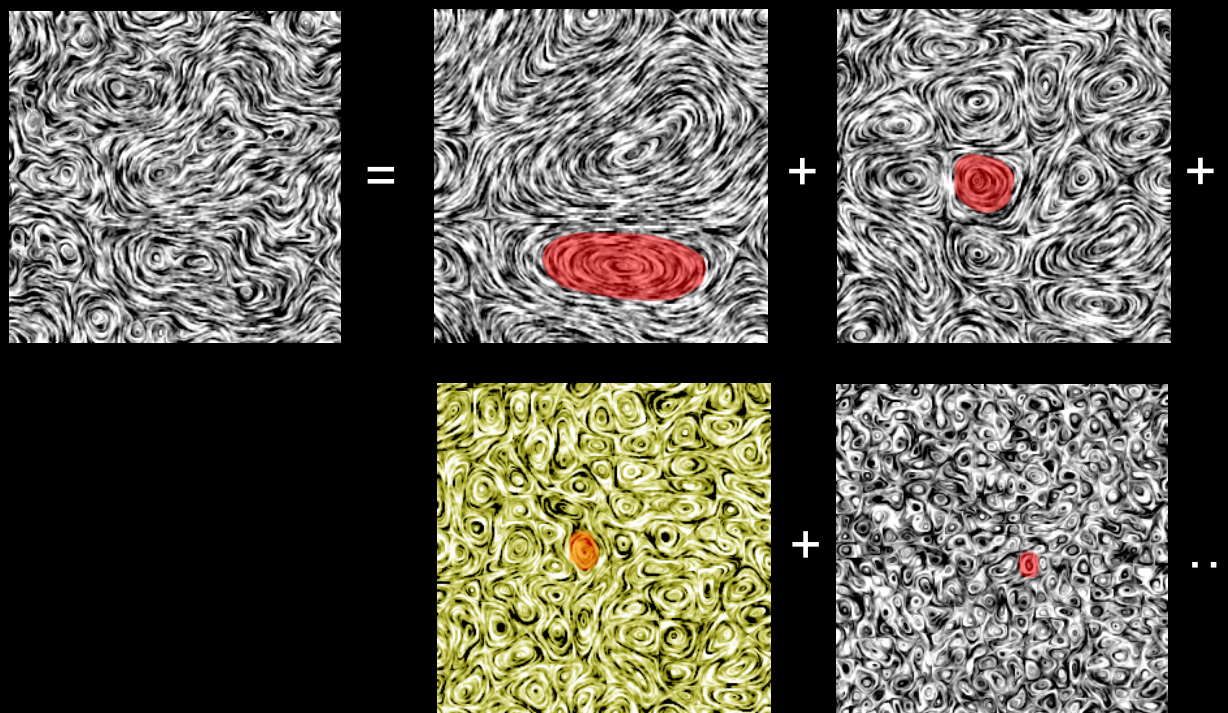
...

Frequency Decomposition

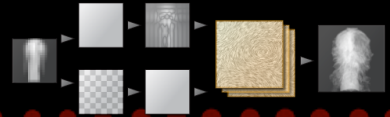


Frequency Decomposition





Energy Spectrum

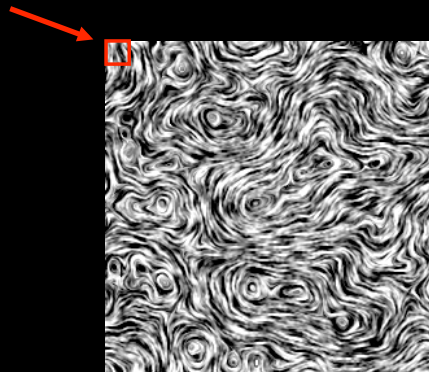


Energy of one grid cell:

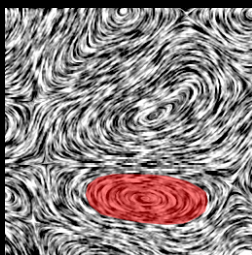
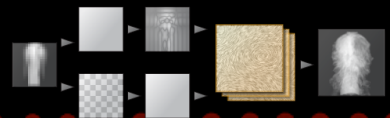
$$e_{x,y} = \frac{1}{2} |v_{x,y}|^2$$

Total energy of grid:

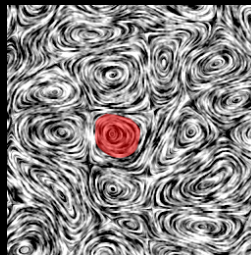
$$E = \sum_{x,y} e_{x,y}$$



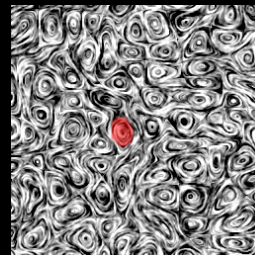
Energy Spectrum



$$E_1 = \sum_{x,y} e_{x,y}$$



$$E_2 = \sum_{x,y} e_{x,y}$$



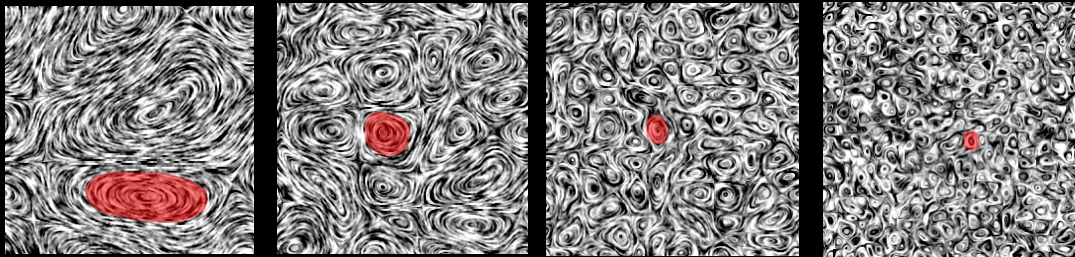
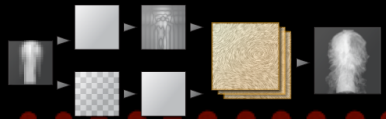
$$E_3 = \sum_{x,y} e_{x,y}$$



$$E_4 = \sum_{x,y} e_{x,y}$$

...

Energy Spectrum

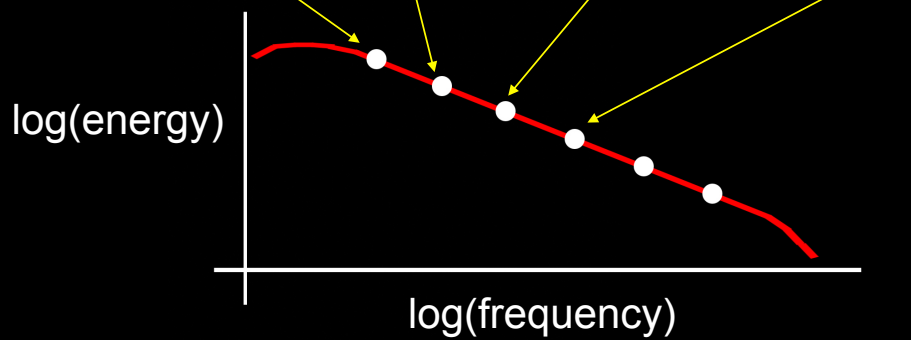


$$E_1 = \sum_{x,y} e_{x,y}$$

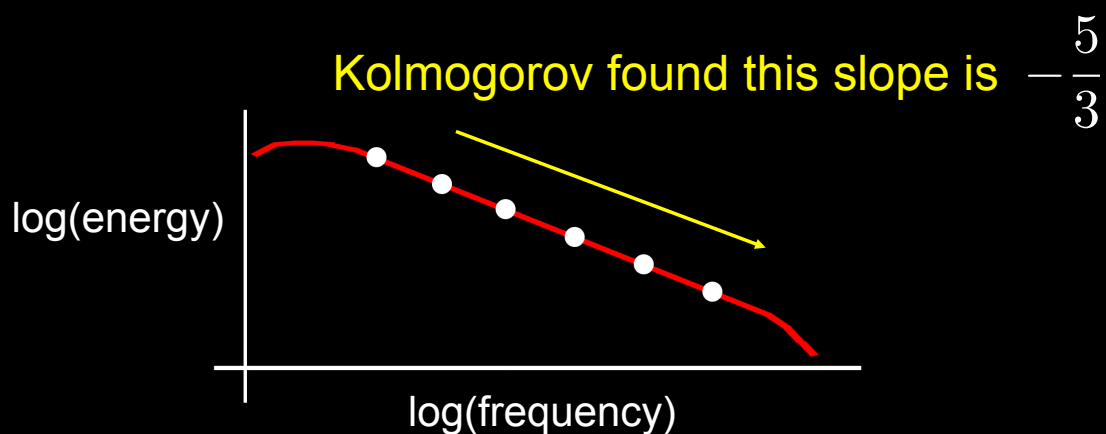
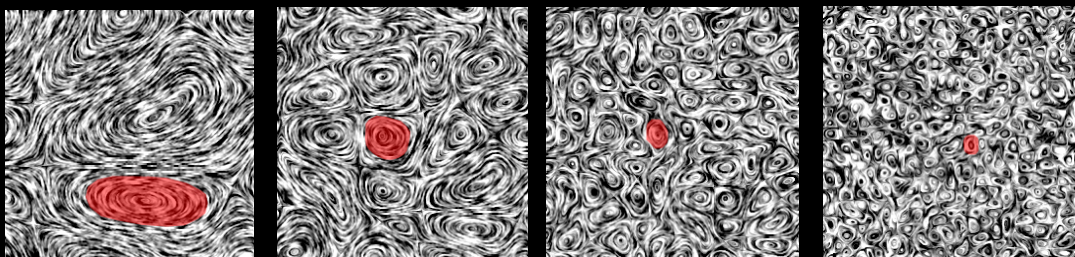
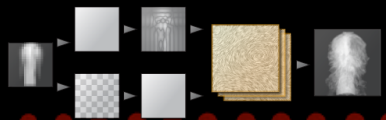
$$E_2 = \sum_{x,y} e_{x,y}$$

$$E_3 = \sum_{x,y} e_{x,y}$$

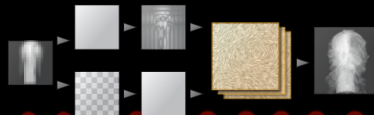
$$E_4 = \sum_{x,y} e_{x,y}$$



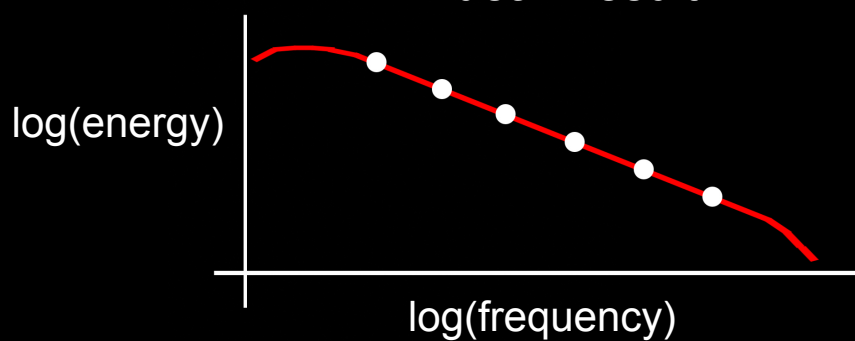
Energy Spectrum



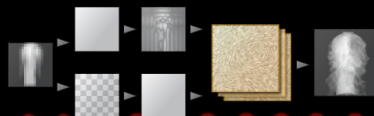
Why Does This Matter?



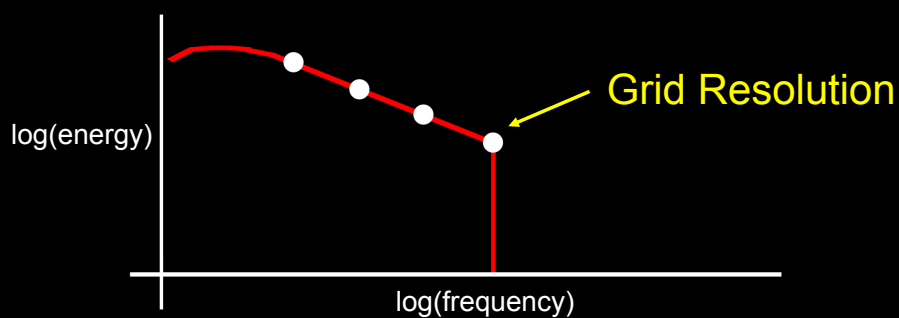
Ideal Result



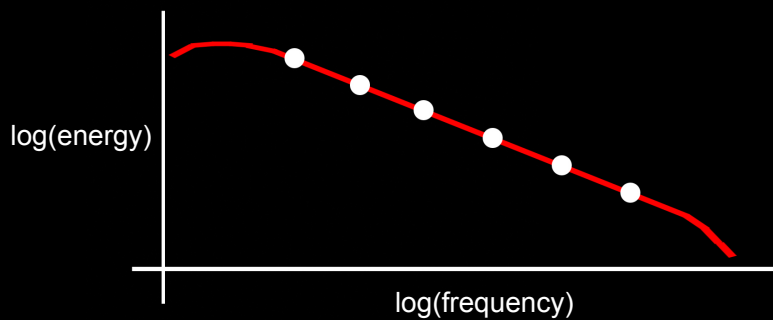
Why Does This Matter?



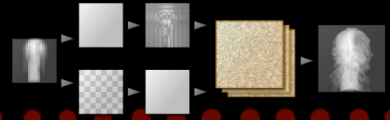
Simulation Result



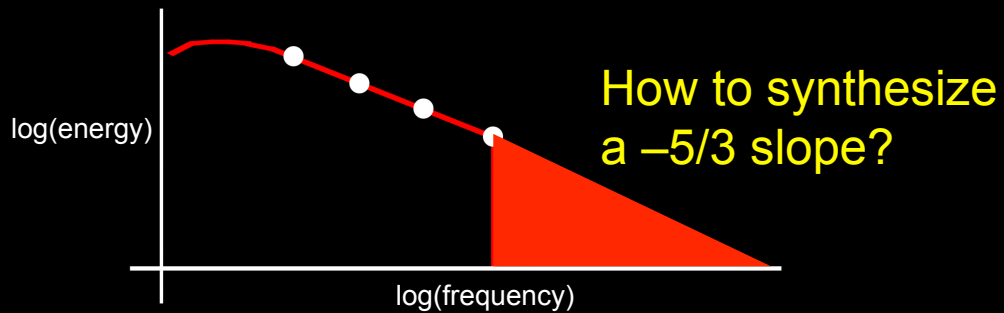
Ideal Result



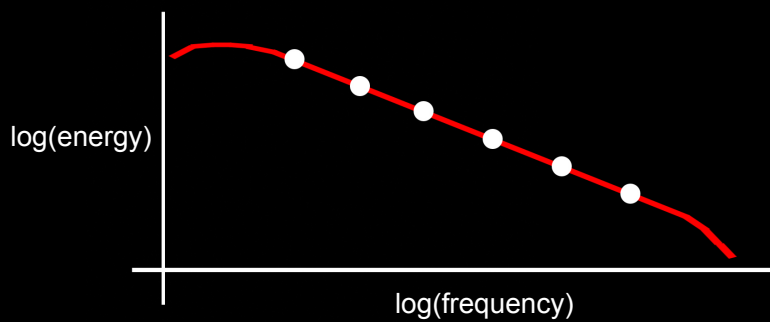
Why Does This Matter?



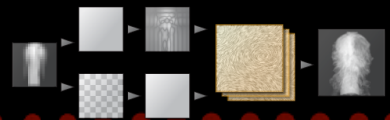
Simulation Result



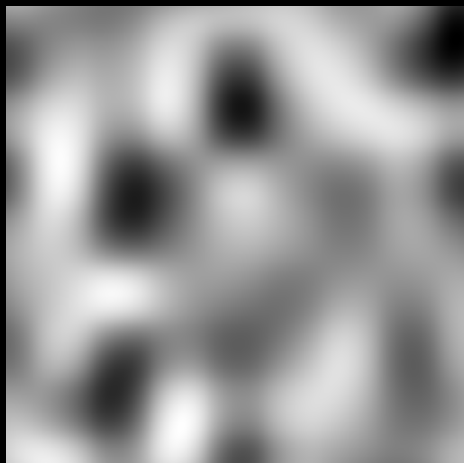
Ideal Result



Spectral Synthesis

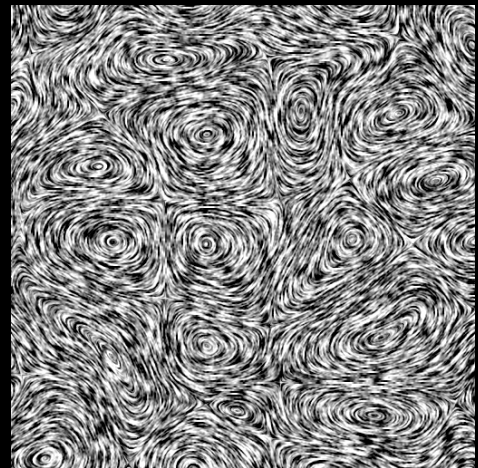


- Use *Wavelet Noise* [Cook and DeRose 2005]



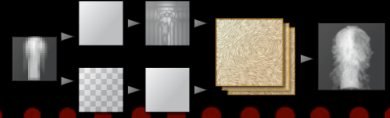
$$\omega(\mathbf{x})$$

→
[Bridson et al. 2007]



$$W(\mathbf{x}) = \nabla \times \omega(\mathbf{x})$$

Spectral Synthesis



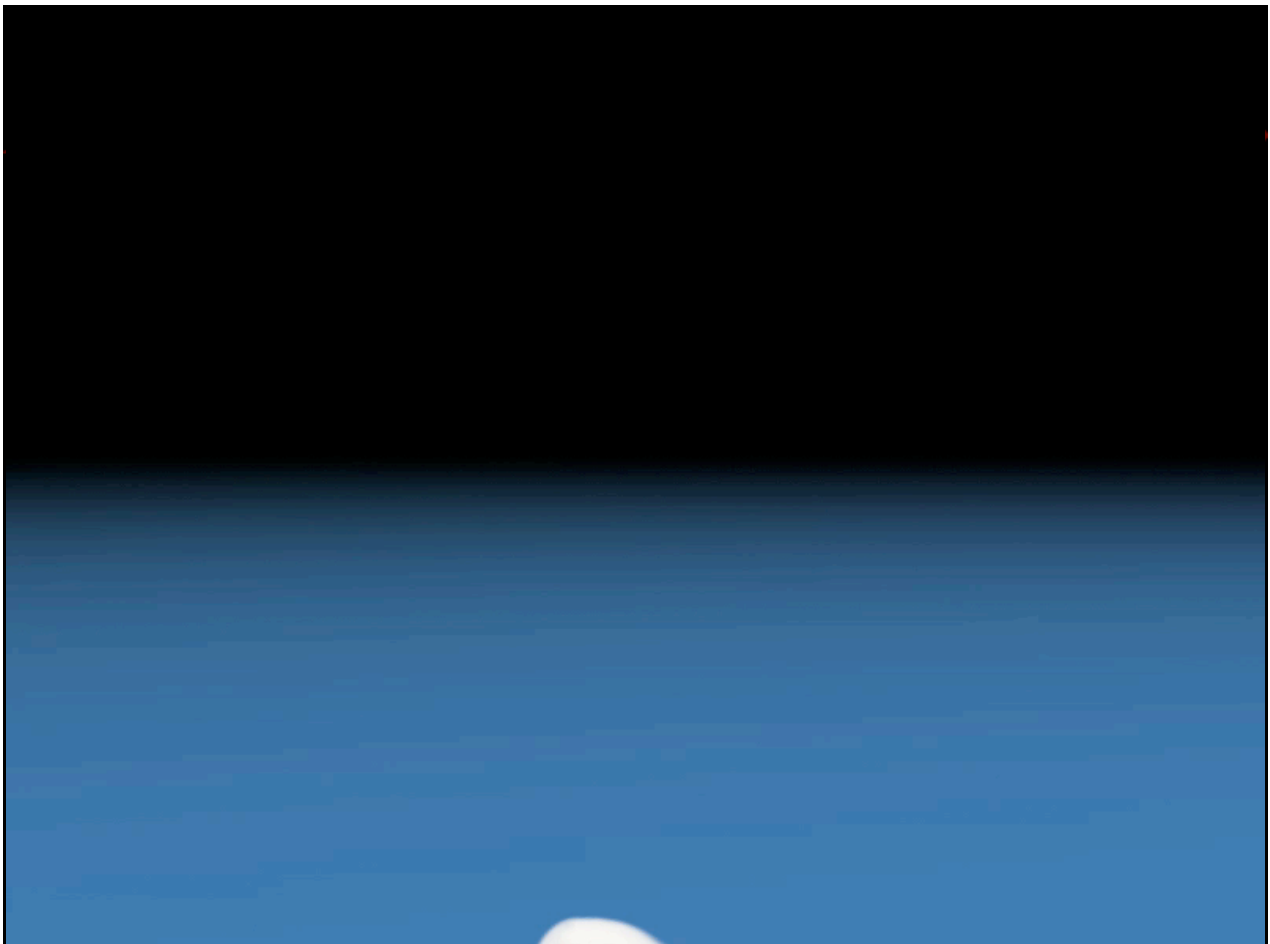
- Perlin *scalar* turbulence:

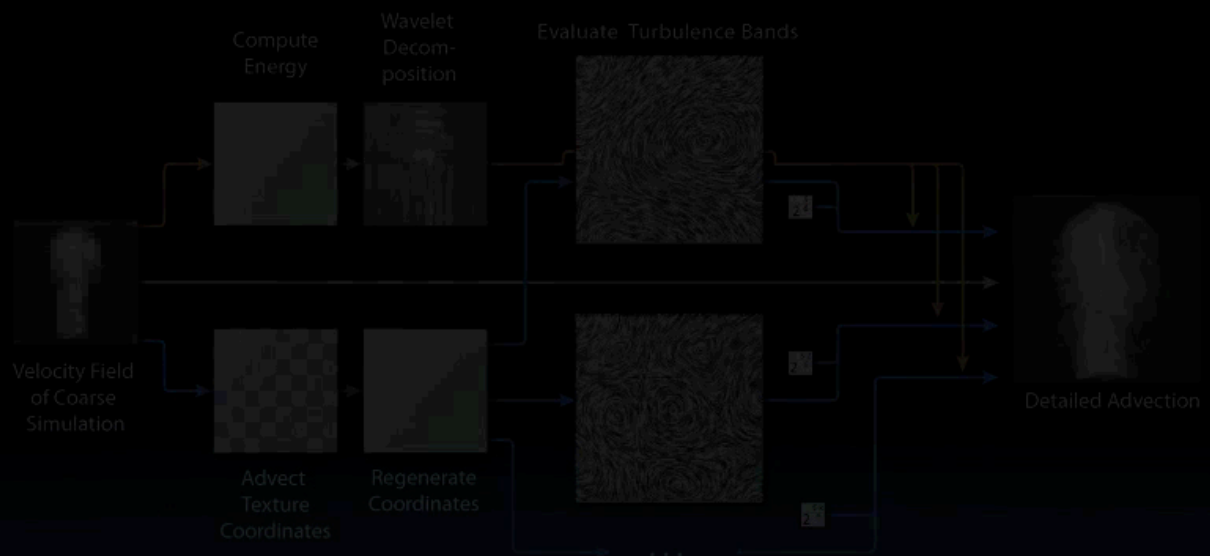
$$turbulence(\mathbf{x}) = \sum_{k=0}^n noise(2^k \mathbf{x}) \cdot 2^{-k}$$

- Wavelet *vector* turbulence:

$$y(\mathbf{x}) = \sum_{k=0}^n W(2^k \mathbf{x}) \cdot 2^{-\frac{5}{6}k}$$

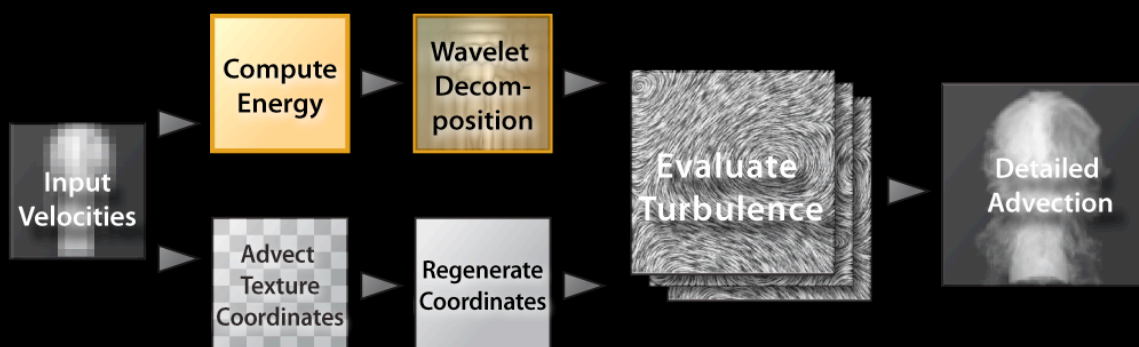
(details in paper)



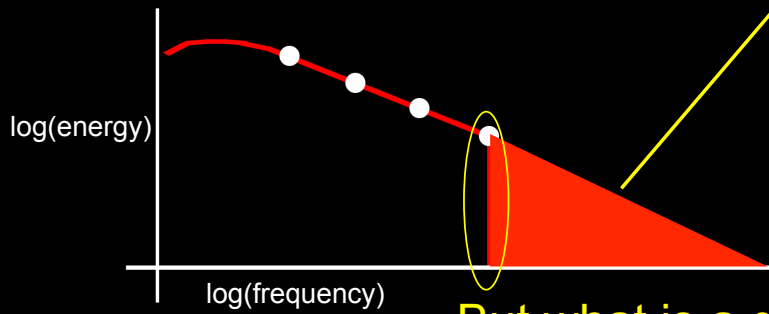
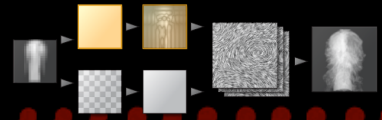


Overview of the Wavelet Turbulence Algorithm

Overview



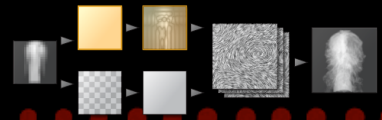
Two Questions



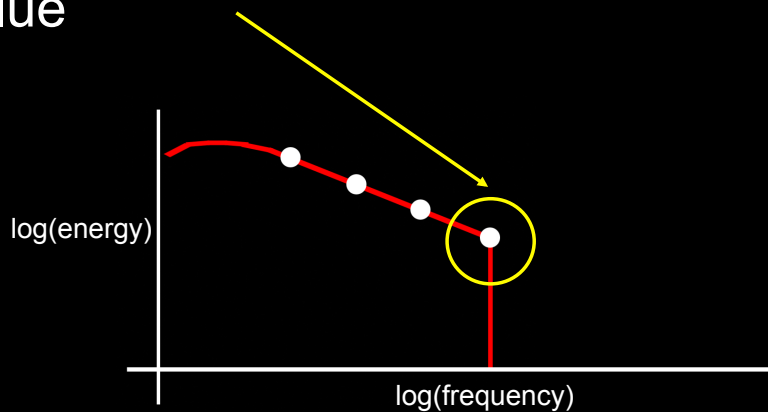
We can synthesize this.

But what is a good coupling here?

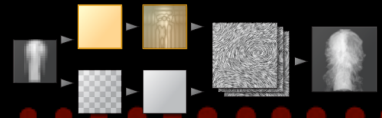
Uniform Weighting



- First attempt: Weight globally with last known value



Uniform Weighting



- Similar to [Stam and Fiume 1993]:

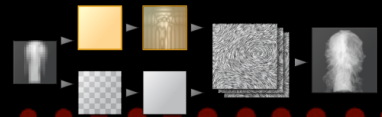
Existing flow:



Add vortices:

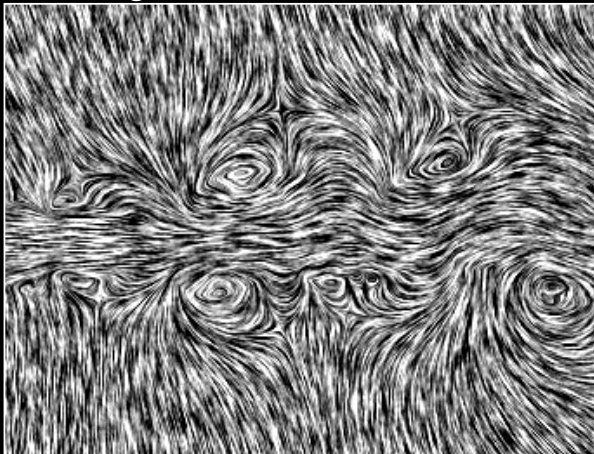


Uniform Weighting



- Similar to [Stam and Fiume 1993]:

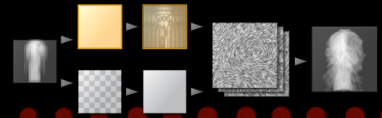
Existing flow:



Add vortices:

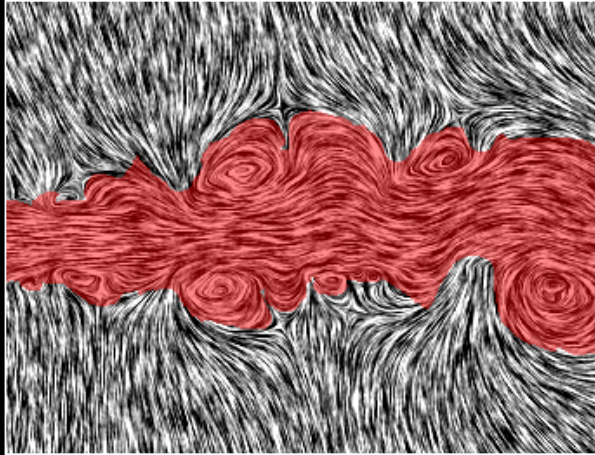


Uniform Weighting

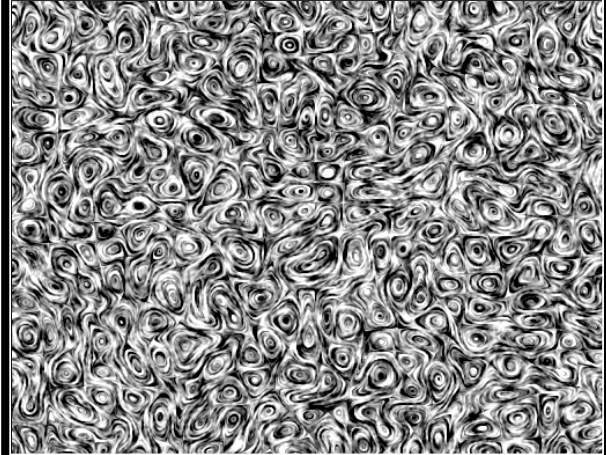


- Similar to [Stam and Fiume 1993]:

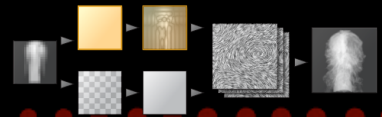
Existing flow:



Add vortices:



Uniform Weighting



- Similar to [Stam and Fiume 1993]:

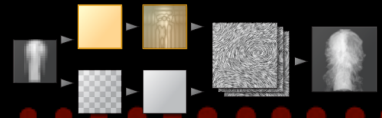
Existing flow:



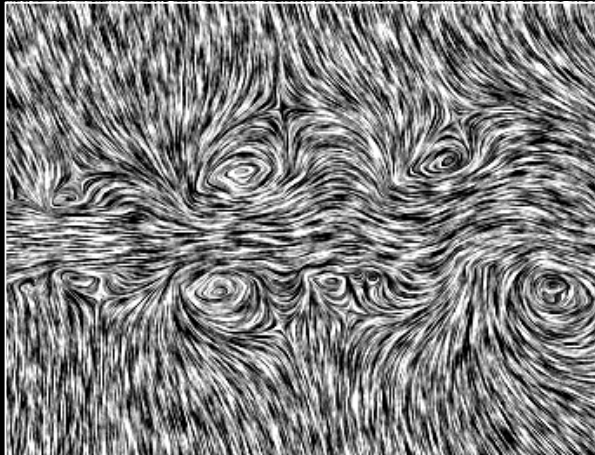
Add vortices:



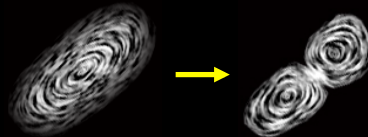
Forward Scattering



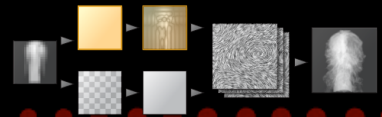
Where in this flow:



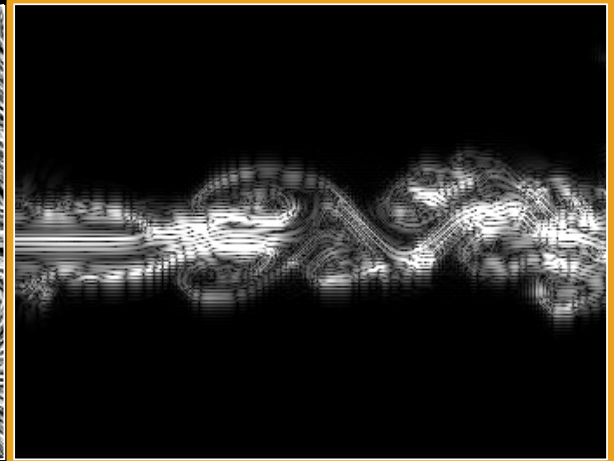
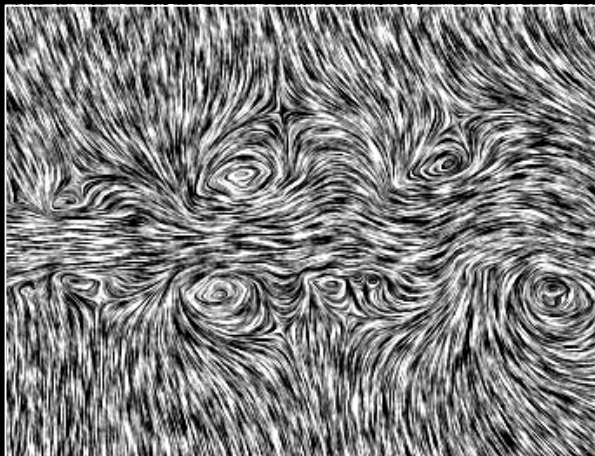
Is this about to happen?



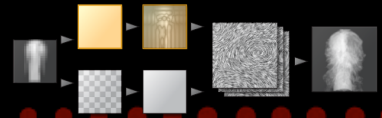
Forward Scattering



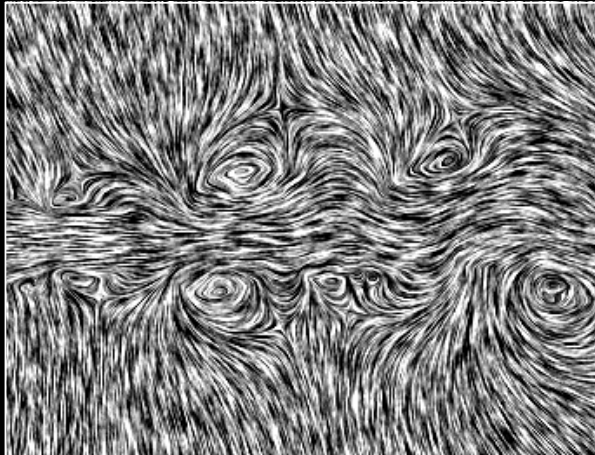
Our solution: highest frequency wavelet component



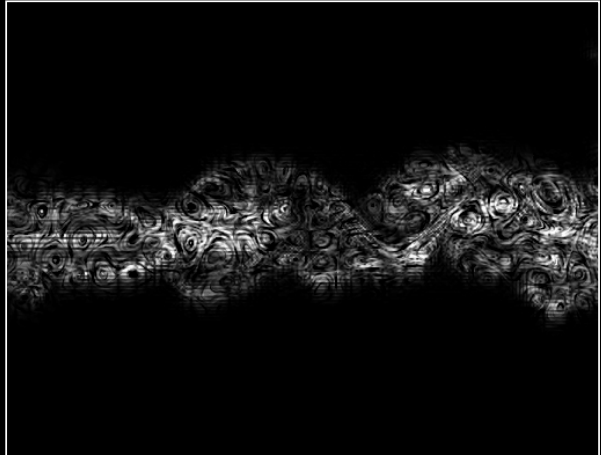
Forward Scattering



Existing flow:

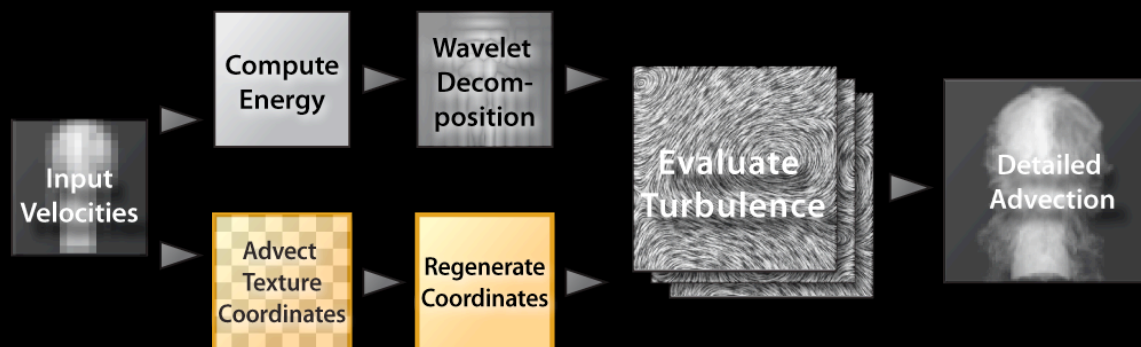


Add vortices:

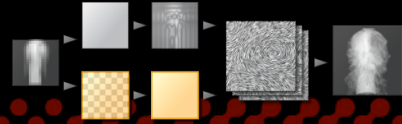




Overview



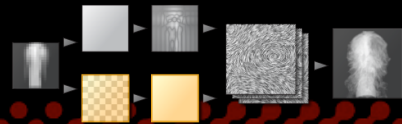
Texture Advection



- Advection of texture coord's
- Contradicting goals:
 - Either move texture with fluid
 - Or prevent distortions
- Compromise necessary...



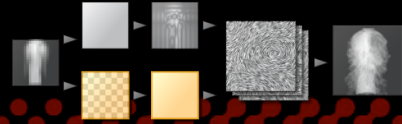
Texture Advection



- Advection of texture coord's
- Contradicting goals:
 - Either move texture with fluid
 - Or prevent distortions
- Compromise necessary...
- Blending: smoothes out / loses detail
See, e.g., [Stam 1999], [Neyret 2003]
- Here: focus on preserving frequencies



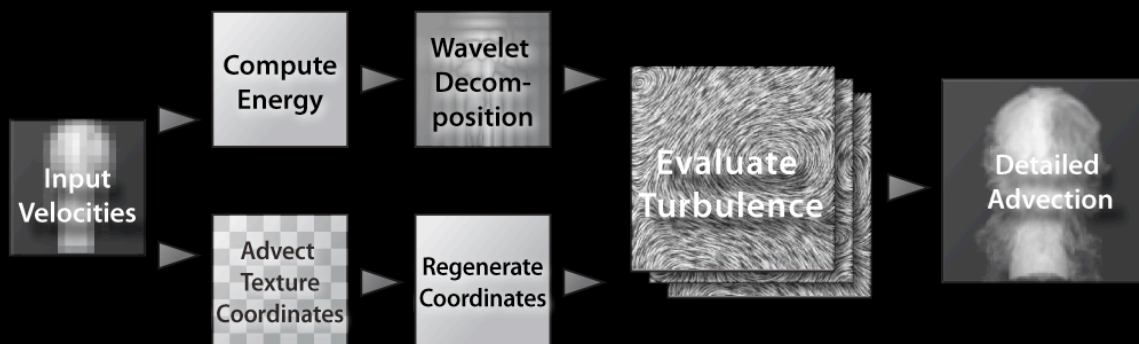
Texture Advection

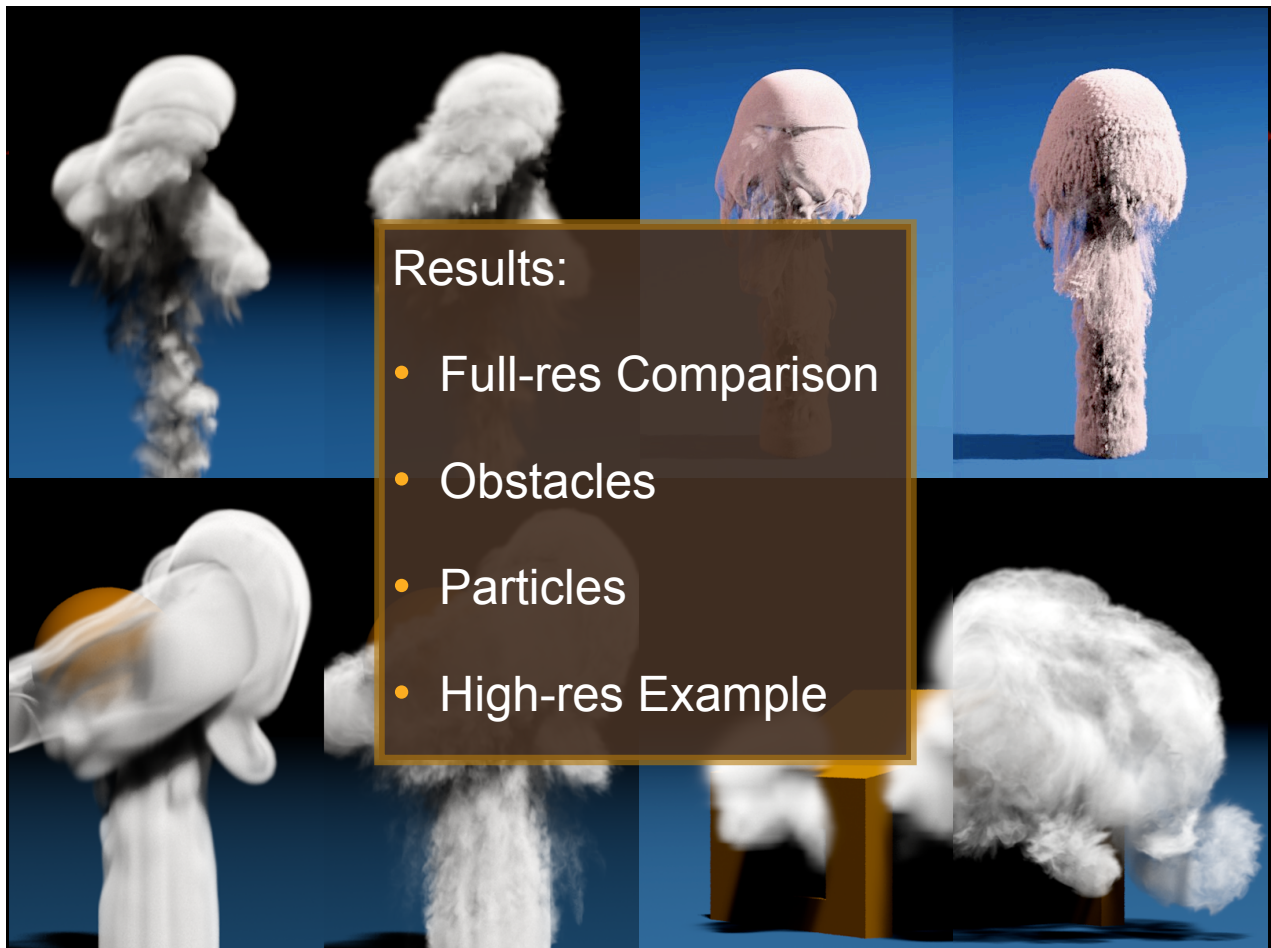


- Advect texture coordinates on grid
- Measure deformations
- Prevent distortions
- Regenerate where necessary

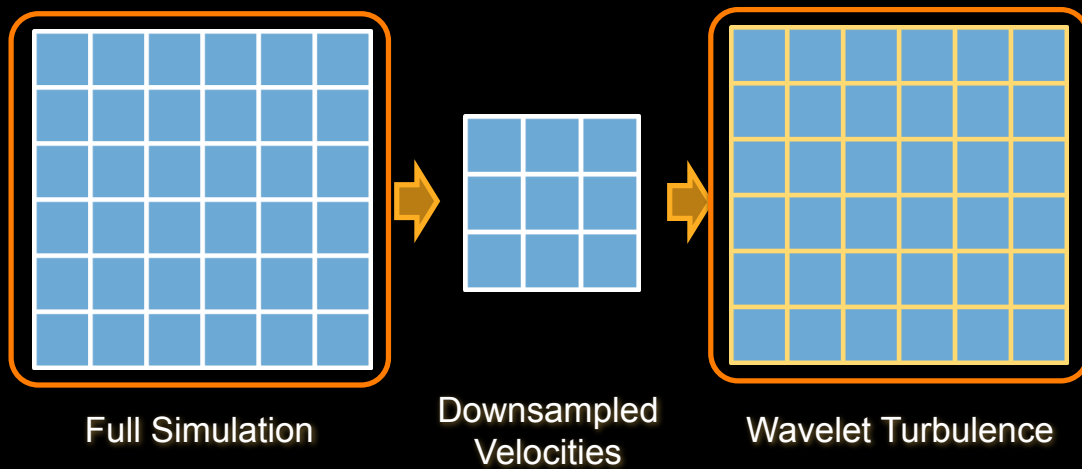


Overview





Results – Comparison to Full Resolution



Results – Comparison to Full Resolution



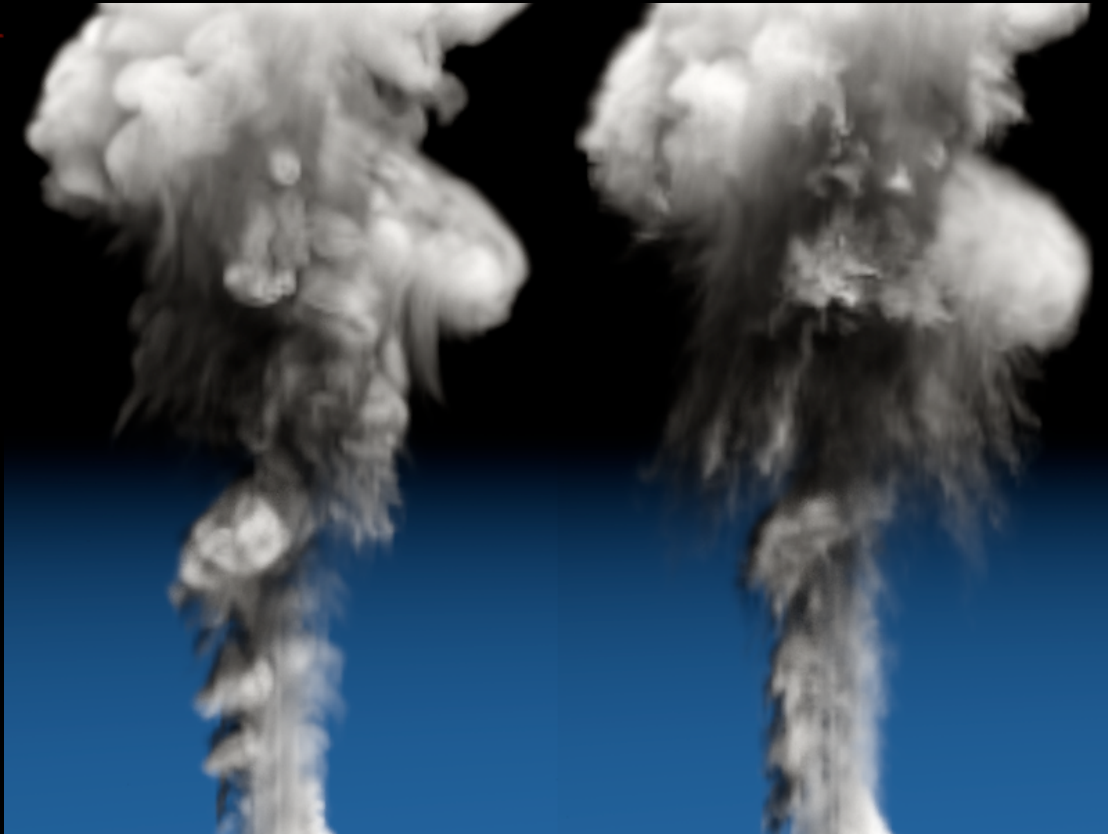
Full Resolution Simulation

Results – Comparison to Full Resolution



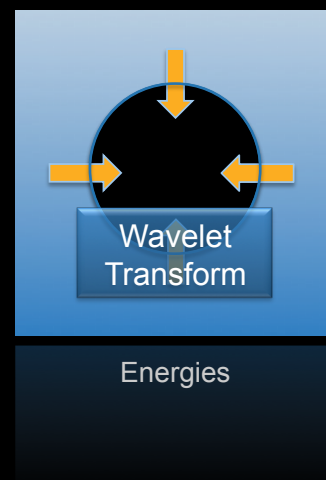
Wavelet Turbulence added to downsampled Velocities

Results – Comparison to Full Resolution



Results - Obstacles

- Obstacle handling:
 - Flow respects obstacle from simulations
 - Extrapolate energies into obstacle region
 - Prevents jumps in the wavelet coefficients

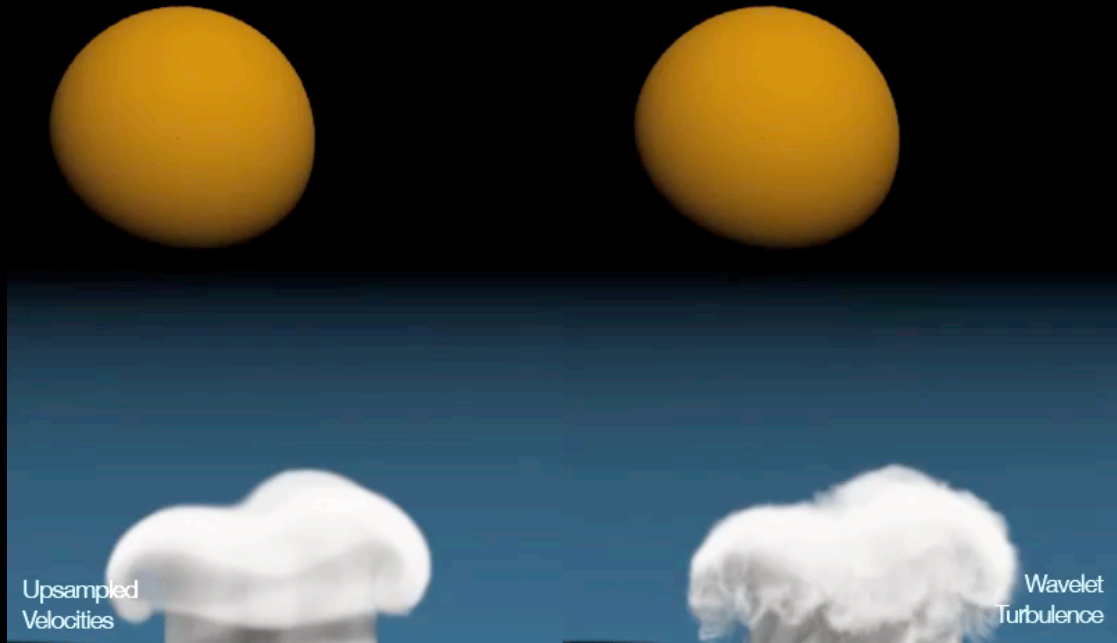


Results - Obstacles

Side by Side Comparison - Flow around an Obstacle

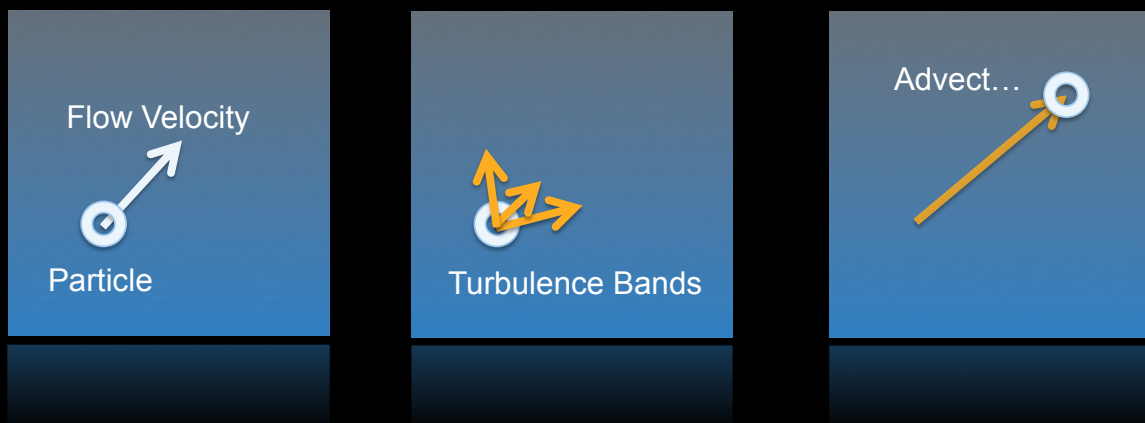
Resolution: 400^3

Underlying Fluid Simulation: 50^3

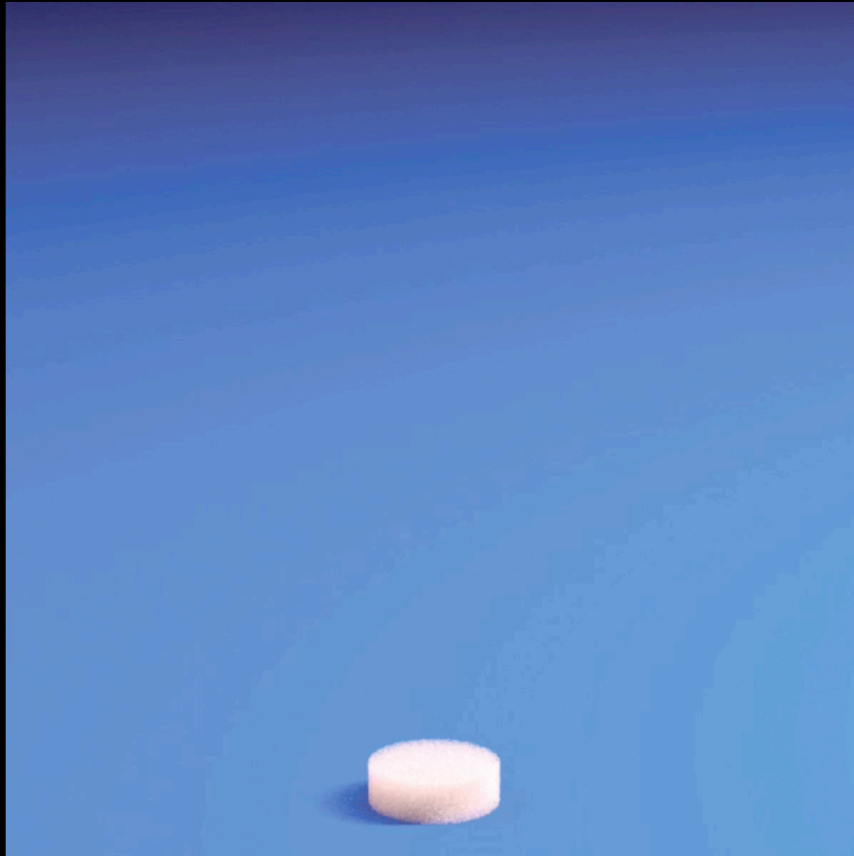


Results - Particles

- No grid necessary
- Evaluate arbitrary numbers of bands



Results - Particles



Results - Control

- Spatial functions

- Example 1: Interval



- Example 2: Ramp



Results - Control

Controlled Wavelet Turbulence
Control Function $v(x)=1$ in two Intervals



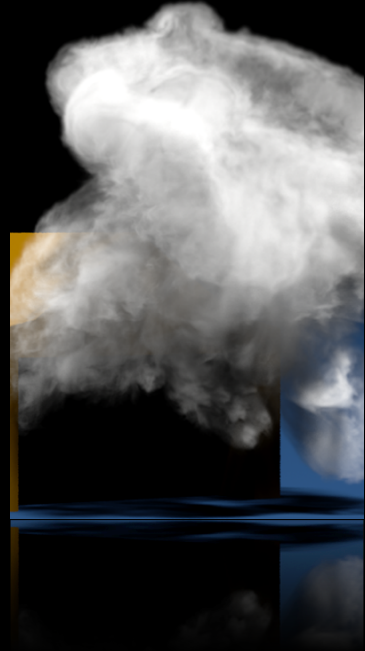
Results – High-res Grid

Side by Side Comparison - High Resolution Grid
Resolution: 22x16x75
Underlying fluid simulation: Navier-Stokes



Wavelet Turbulence - Performance

- Instead of 24+ full size float arrays:
 - 7 for MacCormack step
 - 2 for semi-Lagrange step
 - 0 for particles
- Parallel performance
 - 3.7x speed-up (4 cores)
- Runtime high-res example
 - On average 2 minutes per frame (all steps)



Wavelet Turbulence - Conclusions

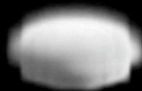
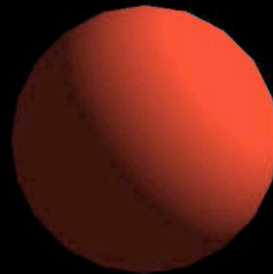
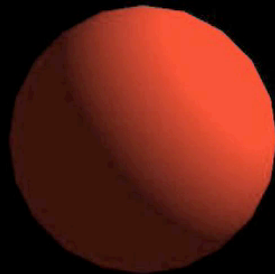
- Add physical detail as post-processing
- Preserves vortex frequencies
- Fast, efficient, low memory
- Relatively simple to implement
- Limitations
 - Cannot reproduce “correct” high-res solution
 - Obstacle interaction depends on low resolution
 - Vortex advection limited due to regeneration

Full Sources
Available Online!



Outlook

- Other
 - F
 - C
- Real



Wavelet Turbulence 240^3 , 4 FPS. Implementation: B. Fierz

Thanks!

- Questions...?



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Source code: <http://www.cs.cornell.edu/~tedkim/WTURB>